Distributed Resource Allocation in D2D-Enabled Multi-tier Cellular Networks: An Auction Approach

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Abstract—Future wireless networks are expected to be highly heterogeneous with the co-existence of macrocells and small cells and they will also provide support for device-to-device (D2D) communication. In such muti-tier heterogeneous systems, centralized radio resource allocation and interference management schemes will not be scalable. In this work, we propose an auction-based distributed solution to allocate radio resources in a muti-tier heterogeneous network. We provide the bound of achievable data rate and show that the complexity of the proposed scheme is linear with the number of transmitter nodes and the available resources. The signaling issues (e.g., information exchange over control channels) for the proposed distributed solution is also discussed. Numerical results show the effectiveness of the proposed solution in comparison with an optimal centralized resource allocation scheme.

Index Terms—Multi-tier cellular networks, device-to-device (D2D) communication, distributed resource allocation, auction method.

I. INTRODUCTION

The future generation (i.e., 5G) of cellular wireless networks are expected to be a mixture of network tiers of different sizes, transmit powers, backhaul connections, different radio access technologies that are accessed by an unprecedented numbers of heterogeneous wireless devices [1]. The multi-tier cellular architecture of 5G networks, where small cells (e.g., femto and pico cells) are underlaid on the macrocells and also there is provisioning for device-to-device (D2D) communications, is expected to not only increase the coverage and capacity of the cell but also improve the broadband user experience within the cell. Since uncoordinated power and spectrum allocation for small cells as well as D2D user equipments (DUEs) can cause severe interference to the other receiving nodes, efficient resource allocation and interference management is one of the fundamental research challenges for such multi-tier heterogeneous networks. Due to the nature of the resource allocation problem in multi-tier cellular networks, centralized solutions are computationally expensive and also incur huge signaling overhead. Therefore, distributed or semi-distributed solutions with low signaling overhead are desirable where the network nodes (such as small cell base stations [SBSs] and DUEs) perform resource allocation independently or by the minimal assistance of macro base stations (MBSs).

In this paper, we use the concept of auction from Economics and present a *distributed solution* to the resource allocation problem for a LTE-A based D2D-enabled multi-tier cellular network. The term *distributed* refers to the fact that the

SBSs and the DUEs independently determine the allocation with the minimal assistance of MBS. The auction approach allows us to develop a polynomial time-complexity algorithm which provides near-optimal performance. The auction process evolves with a bidding process, in which unassigned agents (e.g., transmitters) raise the cost and bid for the resources simultaneously. Once the bids from all the agents are available, the resources are assigned to the highest bidder.

Despite the fact that there are ongoing research efforts to address the resource allocation problems for two-tier networks (e.g., [2]–[4]) as well as for D2D communications (e.g., [5], [6]), the distributed solutions for the radio resource allocation problems in a generic D2D-enabled multi-tier scenario has not been studied comprehensively in the literature. In [2], a distributed solution is proposed for a two-tier network using evolutionary game. A utility-based distributed resource allocation scheme for small cell networks is proposed in [3] using interference pricing. In [5] and [6], energy and QoS-aware resource allocation scheme is proposed for D2D communication. Auction theory has been used in the context of wireless resource allocation problems (e.g., [4], [7]–[9]). An auction-based subchannel allocation for OFDMA and multi-hop systems is presented in [7] and [8], respectively. However, the power allocation is not considered. A resource allocation approach for multi-cell OFDMA system using the concept of auction is presented in [9]. The uplink spectrum-sharing resource allocation problem in a sparsely deployed femtocell networks is considered in [4] and the authors proposed an auction-based solution. However, the works in [5]–[9] consider single-tier systems and the works in [2]–[4] do not consider D2D communication.

Different from the above works, we apply the auction method to solve the radio resource allocation problem in a heterogeneous muti-tier network. We consider a multi-tier network consisting a MBS, a set of SBSs (such as pico and femto base stations) and corresponding small cell user equipments (SUEs), as well as DUEs. There is a common set of radio resources (e.g., resource blocks [RBs]) available to the network tiers (e.g., MBS, SBSs, and DUEs). The SUEs and DUEs use the available resources (e.g., RB and power level) in an underlay manner as long as the interference caused to the macro tier (e.g., macro user equipments [MUEs]) remains below a given threshold. The goal of resource allocation is to allocate the available RBs and transmit power levels to the SUEs and DUEs in order to maximize the spectral efficiency
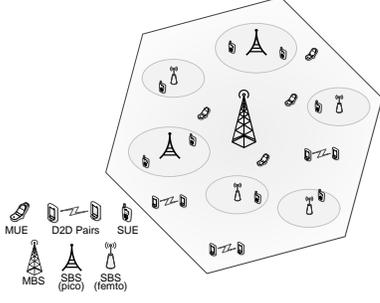
Fig. 1. A D2D-enabled mutil-tier cellular network. The small cells (such as pico and femto cells) as well as the D2D pairs are underlaid within the macrocell reusing the same radio resources.

without causing significant interference to the MUEs.

The main contribution of this work is a low-complexity decentralized solution to the radio resource allocation problem in a multi-tier cellular system. The key feature of the proposed approach is that it incurs polynomial time complexity and low overhead for information exchange. We analyze the complexity and the optimality of the solution. To this end, we also discuss the applicability of the proposed approach in a practical LTE-A based system.

The rest of this paper is organized as follows. The system model, related assumptions, and the resource allocation problem is presented in Section II. The auction-based distributed solution is developed in Section III. We present the numerical results in Section IV before we conclude the paper in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Network Model and Assumptions

Let us consider a transmission scenario in a D2D-enabled multi-tier network as shown in Fig. 1. The network consists of one MBS and a set of M cellular MUEs, i.e., $\mathcal{U}^{\mathrm{m}} = \{1, 2, \cdots, M\}$. There are also D D2D pairs and S closed-access SBSs (such as pico and femto base stations) located within the coverage area of the MBS. The set of SBSs is denoted by $\mathcal{S} = \{1, 2, \cdots, S\}$. We assume that each SBS serves only one SUE during a transmission interval[1]. The set of active SUEs is given by $\mathcal{U}^{\mathrm{s}}$ where $|\mathcal{U}^{\mathrm{s}}| = S$. The set of D2D pairs is denoted as $\mathcal{U}^{\mathrm{d}} = \{1, 2, \cdots, D\}$. In addition, the d-th element of the sets $\mathcal{U}^{\mathrm{d}_T}$ and $\mathcal{U}^{\mathrm{d}_R}$ denotes the transmitter and receiver UE of the D2D pair $d \in \mathcal{U}^{\mathrm{d}}$, respectively. The set of UEs in the network is given by $\mathcal{U} = \mathcal{U}^{\mathrm{m}} \cup \mathcal{U}^{\mathrm{s}} \cup \mathcal{U}^{\mathrm{d}}$. For ease of presentation, we refer to the SBSs (SUEs) and D2D transmitters (receivers) as underlay transmitters (receivers). Therefore, we denote $\mathcal{K}^{\mathrm{T}} = \mathcal{S} \cup \mathcal{U}^{\mathrm{d}_T}$ the set of underlay transmitters (e.g., SBSs and transmitting D2D UEs) and by $\mathcal{K}^{\mathrm{R}} = \mathcal{U}^{\mathrm{s}} \cup \mathcal{U}^{\mathrm{d}_R}$ the set of underlay receivers (e.g., SUEs and receiving D2D UEs). Hence, $K = S + D$ is the total number of underlay transmitters/receivers.

We refer to the small cells and D2D pairs as *underlay tier*. The SBSs and DUEs are underlaid within the *macro tier* (e.g.,

[1]The scheduling of SUEs by the SBSs is not within the scope of this work.

MBS and MUEs) since both the macro tier and the underlay tier (e.g., SBSs, SUEs and D2D pairs) use the same set $\mathcal{N} = \{1, 2, \cdots, N\}$ of orthogonal RBs. In the considered multi-tier network model, each of the network tiers (e.g., macro tier and underlay tier consisting of small cells and D2D UEs) has different transmit power, coverage region, and specific set of users.

Each transmitter node in the underlay tier (e.g., SBS and D2D transmitter) selects one RB from the available N RBs. In addition, the underlay transmitters are capable of selecting the transmit power from a finite set of power levels, i.e., $\mathcal{L} = \{1, 2, \cdots, L\}$. Each SBS and D2D transmitter should select a suitable RB-power level combination. This RB-power level combination is referred to as *transmission alignment* [2]. For each RB $n \in \mathcal{N}$, there is a predefined threshold $I_{\mathrm{TH}}^{(n)}$ for maximum aggregated interference caused by the underlay tier to the macro tier. We assume that value of $I_{\mathrm{TH}}^{(n)}$ is known to the underlay transmitters by using the feedback control channels. An underlay transmitter is allowed to use the particular transmission alignment as long as the cross-tier interference to the MUEs is within the threshold limit.

We assume that the user association to the base stations (either MBS or SBSs) is completed prior to resource allocation. In addition, the potential DUEs are discovered during the D2D session setup by transmitting known synchronization or reference signals (i.e., beacons) [10]. According to our system model, only one MUE is served on each RB to avoid co-tier interference within the macro tier. However, multiple underlay UEs (e.g., SUEs and DUEs) can reuse the same RB to improve the spectrum utilization. This reuse cause severe cross-tier interference to the MUEs, and also co-tier interference within the underlay tier. Therefore, an efficient resource allocation scheme will be required.

B. Achievable Data Rate

The MBS transmits to the MUEs using a fixed power $p_{\mathrm{m}}^{(n)} > 0$ for $\forall n$. For each underlay transmitter $k \in \mathcal{K}^{\mathrm{T}}$, the transmit power over the RBs is determined by the vector $\mathbf{P}_{\mathrm{k}} = \left[p_k^{(1)}, p_k^{(2)}, \cdots, p_k^{(N)} \right]^{\mathsf{T}}$ where $p_k^{(n)} \geq 0$ denotes the transmit power level of transmitter k over RB n. The transmit power $p_k^{(n)}$, $\forall n$ must be selected from the finite set of power levels $\mathcal{L}$. Note that if the RB n is not allocated to transmitter k, the corresponding power variable $p_k^{(n)} = 0$. Since we assume that each underlay transmitter selects only one RB, only one element in the power vector $\mathbf{P}_{\mathrm{k}}$ is non-zero.

We denote by $g_{i,j}^{(n)}$ the channel gain for the links i and j over RB n. For the SUEs, we denote by u_k the SUE associated to SBS $k \in \mathcal{S}$, and for the DUEs, u_k refers to the receiving D2D UE of the D2D transmitter $k \in \mathcal{U}^{\mathrm{d}_T}$. The received signal-to-interference-plus-noise ratio (SINR) for the any arbitrary SUE or D2D receiver, i.e., $u_k \in \mathcal{K}^{\mathrm{R}}, k \in \mathcal{K}^{\mathrm{T}}$ over RB n is given by $\gamma_{u_k}^{(n)} = \frac{g_{k,u_k}^{(n)} p_k^{(n)}}{g_{\mathrm{m},u_k}^{(n)} p_{\mathrm{m}}^{(n)} + \sum\limits_{k' \in \mathcal{K}^{\mathrm{T}}, k' \neq k} g_{k',u_k}^{(n)} p_{k'}^{(n)} + \sigma^2}$ where $g_{k,u_k}^{(n)}$ is the link gain between the SBS and SUE (e.g., $u_k \in \mathcal{U}^{\mathrm{s}}, k \in \mathcal{S}$) or the link gain between the D2D UEs (e.g., $u_k \in \mathcal{U}^{\mathrm{d}_R}, k \in$

$\mathcal{U}^{\mathrm{d}_T}$), and $g_{\mathrm{m},u_k}^{(n)}$ is the interference gain between the MBS and the UE u_k. In the above SINR expression $\sigma^2 = N_0 W_{\mathrm{RB}}$ where W_{RB} is the bandwidth corresponding to an RB and N_0 denotes the thermal noise. Given the SINR, the data rate of the UE u_k over RB n can be calculated as $R_{u_k}^{(n)} = W_{\mathrm{RB}} \log_2 \left(1 + \gamma_{u_k}^{(n)}\right)$.

C. Formulation of the Resource Allocation Problem

The objective of radio resource (i.e., RB and transmit power) allocation problem is to obtain the assignment of RB and power level (e.g., transmission alignment) for the underlay UEs (e.g., D2D UEs and SUEs) that maximizes the achievable sum data rate. The RB and power level allocation indicator for any underlay transmitter $k \in \mathcal{K}^{\mathrm{T}}$ is denoted by a binary decision variable $x_k^{(n,l)}$, where

$$x_k^{(n,l)} = \begin{cases} 1, & \text{if the transmitter } k \text{ is trasnmitting} \\ & \text{over RB } n \text{ with power level } l \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The achievable data rate of an underlay UE u_k with the corresponding transmitter k is written as $R_{u_k} = \sum_{n=1}^{N} \sum_{l=1}^{L} x_k^{(n,l)} W_{\mathrm{RB}} \log_2 \left(1 + \gamma_{u_k}^{(n)}\right)$. The aggregated interference experienced on RB n is given by

$$I^{(n)} = \sum_{k=1}^{K} \sum_{l=1}^{L} x_k^{(n,l)} g_{k,m_k^*}^{(n)} p_k^{(n)} \quad (2)$$

where $m_k^* = \underset{m}{\mathrm{argmax}}\ g_{k,m}^{(n)},\ \forall m \in \mathcal{U}^{\mathrm{m}}$. The concept of reference user [11] is adopted to calculate the aggregated interference $I^{(n)}$ using (2). For any RB n, the interference caused by the underlay transmitter k is determined by the highest gains between the transmitter k and MUEs, e.g., the MUE m_k^* who is the mostly affected UE by the transmitter k. Satisfying the interference constraints considering the link gain corresponding to the reference user will also satisfy the interference constraints for other MUEs. As mentioned earlier, an underlay transmitter is allowed to use a particular transmission alignment only when it does not violate the interference threshold to the MUEs, i.e., $I^{(n)} < I_{\mathrm{TH}}^{(n)},\ \forall n$.

Let $\mathbf{X} = \left[x_1^{(1,1)}, \cdots, x_1^{(1,L)}, \cdots, x_1^{(N,L)}, \cdots, x_K^{(N,L)}\right]^{\mathsf{T}}$ denote the resource (e.g., transmission alignment) allocation vector. Mathematically, the resource allocation problem can be expressed as follows:

(**P1**)
$$\max_{\mathbf{X}} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{l=1}^{L} x_k^{(n,l)} W_{\mathrm{RB}} \log_2 \left(1 + \gamma_{u_k}^{(n)}\right)$$

subject to
$$\sum_{k=1}^{K} \sum_{l=1}^{L} x_k^{(n,l)} g_{k,m_k^*}^{(n)} p_k^{(n)} < I_{\mathrm{TH}}^{(n)},\ \ \forall n \quad (3a)$$

$$\sum_{n=1}^{N} \sum_{l=1}^{L} x_k^{(n,l)} = 1, \qquad \forall k \quad (3b)$$

$$x_k^{(n,l)} \in \{0,1\},\ \forall k,n,l \quad (3c)$$

where

$$\gamma_{u_k}^{(n)} = \frac{g_{k,u_k}^{(n)} p_k^{(n)}}{g_{\mathrm{m},u_k}^{(n)} p_{\mathrm{m}}^{(n)} + \sum_{\substack{k' \in \mathcal{K}^{\mathrm{T}}, \\ k' \neq k}} \sum_{l'=1}^{L} x_{k'}^{(n,l')} g_{k',u_k}^{(n)} p_{k'}^{(n)} + \sigma^2}. \quad (4)$$

The objective of the resource allocation problem **P1** is to maximize the data rate of the SUEs and DUEs. With the constraint in (3a), the aggregated interference caused to the MUEs by the underlay transmitters on each RB is limited by a predefined threshold. The constraint in (3b) indicates that the number of RB selected by each underlay transmitter should be one and each transmitter can only select one power level at each RB. The binary indicator variable for transmission alignment selection is represented by the constraint in (3c). Note that the decision variable $x_k^{(n,l)} = 1$ implies that $p_k^{(n)} = l$.

Corollary 1. *The resource allocation problem* **P1** *is a combinatorial non-convex non-linear optimization problem. The complexity to solve the above problem using exhaustive search is of $\mathcal{O}\left((NL)^K\right)$ and the centralized solution is strongly NP-hard for large values of K, N, L.*

Due to mathematical intractability of solving the above resource allocation problem, in the following we present a distributed solution using tools from auction theory. The distributed solution is developed under the assumption that the system is feasible, i.e., given the resources and parameters (e.g., size of the network, interference thresholds etc.), it is possible to obtain an allocation that satisfies all the constraints of the original optimization problem.

III. DISTRIBUTED SOLUTION USING AUCTION METHOD

The resource allocation using auction is based on the bidding procedure, where the agents (i.e., underlay transmitters) bid for the resources (e.g., RB and power level). The transmitters select the bid for the resources based on the costs (e.g., the interference caused to the MUEs) of using the resource. The desired assignment relies on the appropriate selection of the bids. The unassigned transmitters simultaneously raise the cost of using resource and bid for the resources. When the bids from all the transmitters are available, the resources are assigned to the highest bidder.

In an auction-based assignment model, every resource j associated with a cost c_j and each agent i can obtain benefit B_{ij} from the resource j. The net value (e.g., utility) that an agent i can obtain from resource j is given by $B_{ij} - c_j$. For an equilibrium assignment, every agent i should be assigned with resource j' such that the condition $B_{ij'} - c_{j'} \geq \max_{j} \{B_{ij} - c_j\} - \epsilon$ is satisfied for all the agents, where $\epsilon > 0$ indicates the parameter related to the minimum bid requirement [12]. In the following we utilize the concept of auction in order to obtain the distributed solution of the resource allocation problem.

A. Utility and Cost Function

Let us introduce the parameter $\Gamma_{u_k}^{(n,l)} \triangleq \gamma_{u_k|p^{(n)}=l}^{(n)}$ that denotes the achievable SINR of the UE u_k over RB n using power level l (e.g., $p_k^{(n)} = l$) where $\gamma_k^{(n)}$ is given by (4). We express the data rate as a function of SINR. In particular, let $\mathcal{R}\left(\Gamma_{u_k}^{(n,l)}\right) = W_{\mathrm{RB}} \log_2 \left(1 + \Gamma_{u_k}^{(n,l)}\right)$ denote the achievable data rate for transmitter k over RB n using power level l.

The utility of an underlay transmitter for a particular transmission alignment is determined by two factors, i.e., the achievable data rate for a given RB and power level combination, and an additional cost function that represents the aggregated interference caused to the MUEs on that RB. In particular, the utility of the underlay transmitter k for a given RB n and power level l is given by

$$\mathfrak{U}_k^{(n,l)} = \nu_1 \mathcal{R}\left(\Gamma_{u_k}^{(n,l)}\right) - \left[\nu_2\left(I^{(n)} - I_{\mathrm{TH}}^{(n)}\right)\right]^+ \quad (5)$$

where the operator $[\cdot]^+ = \max\{0, \cdot\}$ and ν_1, ν_2 are the biasing factors which can be selected based on which network tier (i.e., macro or underlay tier) should be given priority for resource allocation [2]. Note that the term $\nu_2\left(I^{(n)} - I_{\mathrm{TH}}^{(n)}\right)$ in (5) represents the cost (e.g., interference caused by underlay transmitters to the MUE) of using RB n. In particular, when the transmitter k is transmitting with power level l, the cost of using RB n can be represented by

$$c_k^{(n,l)} = \nu_2\left(I^{(n)} - I_{\mathrm{TH}}^{(n)}\right)$$
$$= \nu_2\left(g_{k,m_k^*}^{(n)} l + \sum_{\substack{k' \in \mathcal{K}^{\mathrm{T}} \\ k' \neq k}} \sum_{l'=1}^L x_{k'}^{(n,l')} g_{k',m_{k'}^*}^{(n)} p_{k'}^{(n)} - I_{\mathrm{TH}}^{(n)}\right). \quad (6)$$

Let the parameter $C_k^{(n,l)} = \left[c_k^{(n,l)}\right]^+$ and accordingly the cost $C_k^{(n,l)} = 0$ only if $I^{(n)} \leq I_{\mathrm{TH}}^{(n)}$. Using the cost term we can represent (5) as $\mathfrak{U}_k^{(n,l)} = B_k^{(n,l)} - C_k^{(n,l)}$, where $B_k^{(n,l)} = \nu_1 \mathcal{R}\left(\Gamma_{u_k}^{(n,l)}\right)$ is the weighted data rate achieved by transmitter k using resource $\{n, l\}$.

B. Update of Cost and Bidder Information

Let $\mathfrak{b}_k^{(n,l)}$ denote the local bidding information available to transmitter k for the resource $\{n, l\}$. At the beginning of the auction procedure, at any time slot t, each underlay transmitter updates the cost as

$$C_k^{(n,l)}(t) = \max_{k' \in \mathcal{K}^{\mathrm{T}}, k' \neq k} \left\{C_k^{(n,l)}(t-1), C_{k'}^{(n,l)}(t-1)\right\}. \quad (7)$$

In addition, the information of highest bidder of the resource $\{n, l\}$ is obtained by

$$\mathfrak{b}_k^{(n,l)}(t) = \mathfrak{b}_{k*}^{(n,l)}(t-1) \quad (8)$$

where $k* = \underset{k' \in \mathcal{K}^{\mathrm{T}}, k' \neq k}{\mathrm{argmax}} \left\{C_k^{(n,l)}(t-1), C_{k'}^{(n,l)}(t-1)\right\}$. When the cost of $\{n, l\}$ is greater than previous iteration and the transmitter k is not the highest bidder, the transmitter needs to select a new transmission alignment, say, $\{\hat{n}, \hat{l}\}$. The

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Algorithm 1 Auction method for any underlay transmitter k

Input: Parameters from previous iteration: an assignment $\mathbf{X}(t-1) = [\mathbf{x}_k(t-1)]_{\forall k}^{\mathsf{T}}$, aggregated interference $\mathbf{I}(t-1) = \left[I^{(n)}(t-1)\right]_{\forall n}^{\mathsf{T}}$, cost of using resources $\mathbf{C}(t-1) = \left[C_k^{(n,l)}(t-1)\right]_{\forall k,n,l}^{\mathsf{T}}$, and the highest bidders of the resources $\mathfrak{B}(t-1) = [\mathfrak{B}_k(t-1)]_{\forall k}^{\mathsf{T}}$ where $\mathfrak{B}_k(\cdot) = \left[\mathfrak{b}_k^{(n,l)}(\cdot)\right]_{\forall n,l}^{\mathsf{T}}$.

Output: The allocation variable $\mathbf{x}_k(t) = \left[x_k^{(n,l)}\right]_{\forall n,l}^{\mathsf{T}}$, updated costs $\mathbf{C}_k(t) = \left[C_k^{(n,l)}(t)\right]_{\forall n,l}^{\mathsf{T}}$, and bidding information $\mathfrak{B}_k(t) = \left[\mathfrak{b}_k^{(n,l)}(t)\right]_{\forall n,l}^{\mathsf{T}}$ at current iteration t for the transmitter k.

1: Initialize $\mathbf{x}_k(t) := \mathbf{0}$.
2: For all the resources $\{n, l\} \in \mathcal{N} \times \mathcal{L}$ update the cost and highest bidder using (7) and (8), respectively.
3: $\{\tilde{n}, \tilde{l}\} :=$ Non-zero entry in $\mathbf{x}_k(t-1)$. /* Resource assigned in previous iteration.*/
4: **if** $C_k^{(\tilde{n}, \tilde{l})}(t) \geq C_k^{(\tilde{n}, \tilde{l})}(t-1)$ **and** $\mathfrak{b}_k^{(\tilde{n}, \tilde{l})}(t) \neq k$ **then**
5:     $\{\hat{n}, \hat{l}\} := \underset{\{n', l'\} \in \mathcal{N} \times \mathcal{L}}{\mathrm{argmax}} \mathfrak{U}_k^{(n', l')}(t)$.
6:     $\mathfrak{I}^{(\hat{n})} := g_{k,m_k^*}^{(\hat{n})} \hat{l} + I^{(\hat{n})}$. /* Estimate interference level */
7:     **if** $\mathfrak{I}^{(\hat{n})} < I_{\mathrm{TH}}^{(\hat{n})}$ **then**
8:         Set $x_k^{(\hat{n}, \hat{l})} := 1$.
9:         Update the highest bidder for the resource $\{\hat{n}, \hat{l}\}$ as $\mathfrak{b}_k^{(\hat{n}, \hat{l})}(t) := k$.
10:         Increase the cost $C_k^{(\hat{n}, \hat{l})}(t) = C_k^{(\hat{n}, \hat{l})}(t-1) + \Delta_k(t-1)$.
11:     **else**
12:         $\mathbf{x}_k(t) := \mathbf{x}_k(t-1)$. /* Keep the assignment unchanged */
13:     **end if**
14: **else**
15:     $\mathbf{x}_k(t) := \mathbf{x}_k(t-1)$. /* Keep the assignment unchanged */
16: **end if**

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transmitter also increases the cost of the new resource $\{\hat{n}, \hat{l}\}$ as $C_k^{(\hat{n}, \hat{l})}(t) = C_k^{(\hat{n}, \hat{l})}(t-1) + \Delta_k(t-1)$, where $\Delta_k(t-1)$ is given by

$$\Delta_k(t-1) = \Lambda(t-1) + \epsilon. \quad (9)$$

In (9), the variable $\Lambda(\cdot) = \underset{\{n', l'\} \in \mathcal{N} \times \mathcal{L}}{\max} \mathfrak{U}_k^{(n', l')}(\cdot) - \underset{\substack{\{n', l'\} \in \mathcal{N} \times \mathcal{L} \\ n' \neq \hat{n}, l' \neq \hat{l}}}{\max} \mathfrak{U}_k^{(n', l')}(\cdot)$ physically represents the difference between the maximum and the second to the maximum utility value. In the case when the transmitter k does not want to be assigned with a new resource, the allocation from the previous iteration will remain unchanged, i.e., $\mathbf{x}_k(t) = \mathbf{x}_k(t-1)$, where $\mathbf{x}_k = \left[x_k^{(n,l)}\right]_{\forall n,l}^{\mathsf{T}}$.

C. Algorithm for Resource Allocation

We outline the auction-based resource allocation approach in **Algorithm 2**. Each transmitter locally executes **Algorithm 1** and obtains a temporary allocation. After execution of **Algorithm 1**, each underlay transmitter k reports to the MBS the local information, e.g., choices for the resources, $\mathbf{x}_k$. Once the information (e.g., output parameters from **Algorithm 1**) from all the transmitters are available to the MBS, the necessary parameters (e.g., input arguments required by **Algorithm 1**) are calculated and broadcast by the MBS. **Algorithm 1** repeats

Algorithm 2 Auction-based resource allocation

Phase I: Initialization
 1: Estimate the channel state information from the previous time slot.
 2: Each underlay transmitter $k \in \mathcal{K}^{\mathrm{T}}$ randomly selects a transmission alignment and reports to the MBS.
 3: MBS broadcasts the assignment of all transmitters, aggregated interference of each RB, the costs and the highest bidders using pilot signals.
 4: Initialize number of iterations $t := 1$.

Phase II: Update
 5: **while** $\mathbf{X}(t) \neq \mathbf{X}(t-1)$ **and** $t < T_{\max}$ **do**
 6:    Each underlay transmitter $k \in \mathcal{K}^{\mathrm{T}}$ locally runs **Algorithm 1** and reports the assignment $\mathbf{x}_k(t)$, the cost $\mathbf{C}_k(t)$, and the bidding information $\mathfrak{B}_k(t)$ to the MBS.
 7:    MBS calculates the aggregated interference vector $\mathbf{I}(t)$; forms the vectors $\mathbf{X}(t)$, $\mathfrak{B}(t)$, and $\mathbf{C}(t)$; and broadcast to the underlay transmitters.
 8:    Update $t := t + 1$.
 9: **end while**

Phase III: Transmission
10: Use the resources (e.g., the RB and power levels) allocated in the final stage of update phase for data transmission.

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in an iterative manner until the allocation variable $\mathbf{X} = [\mathbf{x}_k]_{\forall k}^{\mathsf{T}}$ remains unchanged for two successive iterations.

D. Optimality, Complexity, and Signaling over Control Channels

Proposition 1. *The sum data rate obtained by the distributed auction algorithm is within $K\epsilon$ of the optimal solution.*

Since the proposed scheme satisfies the allocation constraints (3b)-(3c), and also maintains the interference threshold given by (3a), the solution obtained by the auction algorithm gives a lower bound of original resource allocation problem **P1**. As shown in the following proposition, when the allocation remain unchanged for at least $T \geq 2$ consecutive iterations, the complexity of the proposed solution is linear with number of underlay transmitters and the available resources.

Proposition 2. *The auction algorithm converges to a fixed allocation with the number of iterations of $\mathcal{O}(TKNL\Upsilon)$ where $\Upsilon = \left\lceil \dfrac{\max\limits_{k,n,l} B_k^{(n,l)} - \min\limits_{k,n,l} B_k^{(n,l)}}{\epsilon} \right\rceil$.*

The proofs of the above propositions can be found in [13]. In the following, we discuss the applicability of the proposed method in a practical system. In the proposed solution, the MBS only needs the allocation vector $\mathbf{X}$ and the channel gain vector $\mathbf{G} = \left[g_{k,m_k^*}^{(n)} \right]_{\forall k,n}^{\mathsf{T}}$ to calculate the aggregated interference vector, e.g., $\mathbf{I} = \left[I^{(n)} \right]_{\forall n}^{\mathsf{T}}$. Also note that at each iteration, the MBS and the transmitters need to exchange the allocation vector $\mathbf{X}$, information about highest bidders $\mathfrak{B}$, and the cost vector $\mathbf{C}$ for the underlay transmitters. In LTE-A based systems, these information can easily be incorporated into the standard control channel messages. For example, the MBS can broadcast these information vectors using physical downlink control channel (PDCCH). Likewise, each of the underlay transmitters k can inform the MBS the local assignment preferences $\mathbf{x}_k$, updated costs $\mathbf{C}_k$, and bidding information $\mathfrak{B}_k$ using physical hybrid-ARQ indicator channel (PHICH).

IV. NUMERICAL RESULTS

A. Simulation Setup

We develop a MATLAB-based simulator and observe the performance of our proposed approach using simulations. We simulate a 300 m × 300 m area where the MBS is located in the center of the cell and $M = 6$ MUEs are uniformly distributed within the cell radius. The SBSs and SUEs are uniformly distributed within the macro cell and small cell radius, respectively. The DUEs are located according to the clustered distribution model [14]. We obtain the minimum bid increment parameter ϵ by trial and error and set it to 100. We choose the number of RBs $N = 6$, biasing factors $\nu_1 = \nu_2 = 1$, and assume the interference threshold to be -70 dBm for all the RBs.

For modeling the propagation channel, we consider distance-dependent path-loss and shadow fading; furthermore, the channel is assumed to experience Rayleigh fading. For example, the following path-loss equation is used to estimate path-loss between SBSs and SUEs as well as to the MUEs and DUEs: $\psi_{\mathrm{S}}(\ell)_{[dB]} = 38.46 + 20\log(\ell) + \xi_{ss} + 10\log(\varsigma)$. The path-loss between the MBS and MUEs as well as DUEs and SUEs is estimated as $\psi_{\mathrm{M}}(\ell)_{[dB]} = 15.3 + 40\log(\ell) + \xi_{sm} + 10\log(\varsigma) + \xi_w$. Similarly, the direct link gain between DUEs is given by $\psi_{\mathrm{D}}(\ell)_{[dB]} = 148 + 40\log(0.001\ell) + \xi_{sd} + 10\log(\varsigma) + \xi_w$. In the path-loss equations, ℓ is the distance between nodes in meter, ξ_{ss}, ξ_{sm}, ξ_{sd} account for shadow fading and are modeled as log-normal random variables, ς is an exponentially distributed random variable which represents the Rayleigh fading channel power gain, and ξ_w denotes outdoor wall loss.

B. Results

The convergence behavior of the proposed solution is depicted in Fig. 2. We observe that the data rate of the network becomes stable within 100 iterations. In order to observe the convergence behavior in different network density, we vary the number of underlay nodes (e.g., SBSs, SUEs, and DUEs) and plot the empirical cumulative distribution function (CDF) of the number of iterations required for convergence in Fig. 2(b). The empirical CDF is defined as $\widehat{F}_\tau(\jmath) = \frac{1}{\tau} \sum\limits_{i=1}^{\tau} \mathbb{I}_{[\zeta_i \leq \jmath]}$, where τ is the total number of simulation observations, ζ_i is the number of iterations required for convergence at the i-th simulation observation, and $\jmath$ represents the x-axis values in Fig. 2(b). The indicator function $\mathbb{I}_{[\cdot]}$ outputs 1 if the condition $[\cdot]$ is satisfied and 0 otherwise. As mentioned in **Proposition 2**, the convergence to the fixed allocation depends on number of underlay transmitters. When the number of network nodes increases, the number of iterations required for convergence increases. This is because, the underlay transmitters need to execute the algorithm more times in order to obtain the updated bidding and cost information, and hence to find the fixed allocation. However, even in a moderately dense network with 30 underlay nodes (e.g., $S+D = 9+6 = 15$), the solution converges to a fixed rate within 100 iterations.
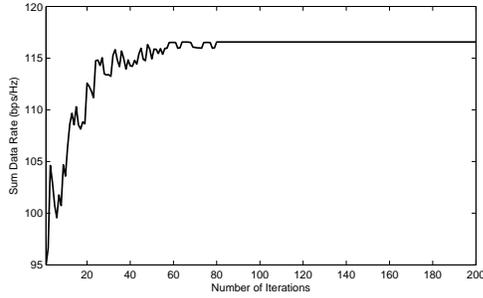
(a)
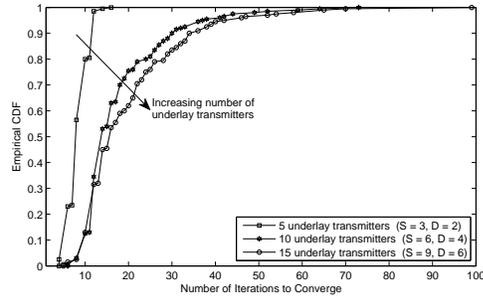
(b)

Fig. 2. (a) The convergence of proposed solution (for $L = 3$, e.g., $\mathcal{L} = \{3, 5, 7\}$ dBm, $S = 6$, $D = 4$), and (b) Empirical CDF of the number of iterations required for convergence using similar power levels as those for Fig. 2(a).
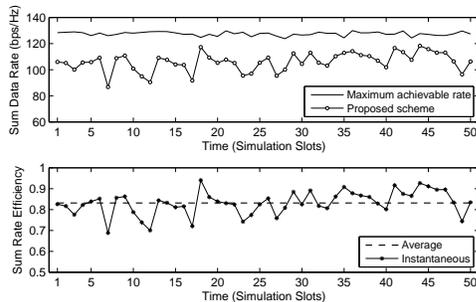

Fig. 3. Achievable sum rate of the proposed scheme and the upper bound. We consider $S = 3$, $D = 2$, and $\mathcal{L} = \{3, 5\}$ dBm.

To evaluate the performance of the proposed algorithm, in Fig. 3 we compare the the data rate with the upper bound obtained from the solution of the original optimization problem **P1**. We obtain the upper bound (e.g., maximum achievable data rate) using exhaustive search. We examine the sum rate of the network in a period of 50 time slots and average the results over different spatial realizations. We measure the sum rate efficiency as $\eta = \frac{R_{\text{prop}}}{R_{\text{max}}}$ where R_{prop} and R_{max} denote the sum data rate obtained from proposed scheme and maximum achievable sum rate, respectively. From this figure and as mentioned in **Proposition 1**, it can be observed that the data rate of the proposed solution is within $K\epsilon$ of the maximum achievable data rate. Recall that the

original resource allocation problem is an NP-hard problem and the computational complexity of exhaustive search to achieve the upper bound is of $\mathcal{O}\left((NL)^K\right)$. The proposed scheme provides data rates which are close to 80% of the maximum achievable data rate, however, with significantly less computational complexity and signaling overhead.

V. CONCLUSION

We have presented a polynomial time-complexity distributed solution for resource allocation in D2D-enabled multi-tier cellular networks. We have analyzed the optimality and complexity the solution. Numerical results have shown that the proposed solution performs close the upper bound of achievable data rate with significantly less complexity and minimal involvement of the central controller node. Capturing the dynamics of misbehaving and selfish transmitter nodes using other auction-based methods (e.g., truthful auction) could be an interesting topic of future research.

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