Distributed Uplink Power Control for Multi-Cell Cognitive Radio Networks

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***Abstract*—We present a distributed power control algorithm to address the uplink interference management problem in cognitive radio networks where the underlaying secondary users (SUs) share the same licensed spectrum with the primary users (PUs) in multi-cell environments. Since the PUs have a higher priority of channel access compared to the SUs, minimal number of SUs should be gradually removed, subject to the constraint that all primary users are supported with their target signal-to-interference-plus-noise ratios (SINRs), which is assumed feasible. In our proposed algorithm, each primary user rigidly tracks its target-SINR by employing the conventional target-SINR tracking power control algorithm (TPC). Each transmitting SU employs the TPC as long as the total received power at the primary receiver is below a given threshold; otherwise, it decreases its transmit power in proportion to the ratio between the given threshold and the total received power at the primary receiver, which is referred to as the *total received-power-temperature*. We show that our proposed distributed power-update function has at least one fixed-point. We also show that our proposed algorithm not only improves the number of supported SUs but also guarantees that all primary users are supported with their (feasible) target-SINRs. Finally, we also propose an enhanced power control algorithm that achieves zero-outage for PUs and a better outage ratio for SUs. To this end, we provide a robust power control method that considers the uncertainties in channel gains.**

***Index Terms*—Cellular cognitive wireless networks, interference temperature limit, underlay and overlay spectrum access, mixed strategy spectrum access, distributed interference control, total received-power-temperature, uncertain channel state information, uncertainty sets.**

I. Introduction

In a cognitive radio network (CRN), secondary users (SUs) coexist with primary users (PUs) using spectrum overlay or spectrum underlay to exploit the radio spectrum licensed to PUs. In the overlay strategy, when a PU is active, no SU transmits (i.e., the interference temperature limit is assumed to be zero), and thus the interference tolerability of the primary network is ignored. On the other hand, in the pure underlay strategy, which uses a fixed interference temperature limit, the transmission opportunities of SUs during the idle periods (i.e., when no PU is active) are wasted [1]. Therefore, instead of assuming that the interference temperature limit is fixed, we propose that it can be varied dynamically in an optimum manner and a *mixed-strategy* can be adopted. In particular, the value of interference temperature at each primary receiver can be dynamically decreased (increased) as the number of its corresponding PUs is increased (decreased) and/or their channel status becomes weaker (stronger). For example, when many PUs with large target-signal-to-interference-plus noise ratio (SINR) requirements are active and/or the corresponding channel gains (from transmitters to the receivers) are poor, the interference temperature is set to a very small value (or even zero). This corresponds to the spectrum overlay strategy. On the other hand, when a number of PUs with moderate target-SINR requirements and/or good channel gains are active, a non-zero value of the interference temperature limit can be chosen such that the requirements of the PUs can still be satisfied. This corresponds to the spectrum underlay strategy.

By dynamically setting the value of the interference temperature limit, a mixed-strategy is obtained. With this mixed strategy, the spectrum access opportunities as well as the interference tolerability of the primary network, which are missed in the pure underlay and overlay strategies, respectively, can be exploited to improve the performance of SUs. This mixed strategy can be implemented through an efficent power control method. However, this power control problem is a non-deterministic polynomial-time (NP)-complete problem [2] and *centralized* algorithms have been proposed in [2], [3] to solve the problem sub-optimally. However, the signalling complexity of such algorithms could be high and these schemes might be useful for benchmarking purpose only.

In this paper, we address the problem of *distributed* uplink power control in cellular CRNs. Having obtained the interference temperature limit of each primary receiver, we aim to devise a distributed power control scheme for the PUs and SUs to set their transmit power levels so that a maximal number of SUs reach their target-SINRs, while all the PUs are supported with their target-SINRs (i.e., the interference caused by the SUs to each primary receiver remains below its interference temperature limit).

The existing distributed interference management algorithms in conventional cellular wireless networks do not guarantee that the total interference caused to PUs by SUs does not exceed a given threshold, which result in outage of some PUs (i.e., some

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PUs are not supported with their required SINRs). However, these algorithms can be used by the SUs, provided that the interference caused by them to the PUs does not exceed a given threshold. In particular, if the SUs limit their transmit power levels so that the total interference caused to the PUs does not exceed a given threshold (which each primary receiver can broadcast to all SUs), each PU is able to reach its target-SINR, and the SUs can minimize their outage ratios by employing an existing distributed power control algorithm. This is the idea that we use in this paper to develop distributed uplink power control algorithms. The contributions of this paper can be summarized as follows.

- We formally define the problem of uplink power control in CRNs in multicellular environments to minimize the outage ratio for the SUs subject to the zero-outage constraint for the PUs. We present a distributed power control scheme to achieve this design goal. Specifically, in our proposed algorithm, each PU rigidly tracks its target-SINR by employing the traditional TPC algorithm proposed in [7]. Each transmitting SU employs the TPC algorithm as long as the total received power at each of the primary receivers is below a given threshold; otherwise, it decreases its transmit power in proportion to the ratio of the given threshold to the total received power at a primary receiver. We refer to our proposed algorithm as TPC with PU-protection (TPC-PP).
- We prove that the proposed distributed power-update function corresponding to TPC-PP has at least one fixed-point. We also show that the proposed algorithm not only significantly decreases the outage ratio of SUs, but also guarantees zero-outage ratio for the PUs.
- We also devise an improved TPC-PP algorithm (called ITPC-PP), which achieves better outage ratios for SUs and zero-outage for PUs.
- Due to the stochastic nature of wireless channels we develop a power control algorithm that is resilient against channel fluctuations. We refer to this algorithm as robust TCP-PP (RTPC-PP). Through simulations we show that the RTPC-PP scheme is robust against channel uncertainties at the cost of a higher outage ratio compared to TPC-PP.
- Performances of the proposed algorithms are evaluated and also compared against a state-of-the-art centralized algorithm for uplink power control for cellular CRNs.

It is worth noting that emerging wireless networks such as the multi-tier cellular networks and/or device-to-device communication networks, face the same problem of prioritized uplink power control and interference management where all users in different tiers share the same licensed spectrum but with different priorities of access. Thus our proposed power control algorithms can also be employed in such networks for cross-tier interference management. Also, note that the proposed power control methods can be used for both orthogonal frequency-division multiple access (OFDMA) and code-division multiple access (CDMA)-based CRNs. While in the former case uplink power control is performed for transmission over different sub-channels shared among PUs and SUs over space and time, in the latter case, uplink power control is performed for transmission over the entire spectrum (i.e., a single channel).

The rest of this paper is organized as follows. Section II reviews the related literature and discusses the motivation and novelty of this work. In Section III, we introduce the system model and existing distributed power control algorithms, and present a formal statement of the interference management problem in CRNs. Section IV introduces our proposed distributed interference control algorithm. In Section V, we analyze the proposed method and derive its key properties. Section VI describes how the proposed algorithm can be improved. The power control algorithm under channel uncertainty is provided in Section VII. Simulation results are presented in Section VIII. Section IX concludes the paper.

II. Related Literature and Novelty of the Work

A few works in the literature have addressed the uplink power control and admission control problem in CRNs (Table I). These works are based on removal algorithms, whereby the least number of SUs are removed so that all admitted SUs obtain their target-SINRs and they do not cause outage to any PU. The removal algorithms generally consist of two phases, namely, the *feasibility checking phase* and the *removal phase*.

To check the feasibility of the constraints, the existing algorithms as in [4] use centralized techniques based on calculating spectral radius of a matrix of path-gains and target-SINRs developed in [5]. The algorithms in [3] and [6] use the TPC algorithm proposed in [7]. In [8], a random searching algorithm is proposed where probabilistic mechanisms are used for the SUs to access the channel. This algorithm may not converge and its performance depends on the initial starting point. In [9] and [3], sequential admission control algorithms are proposed in which, based on certain metrics, an opportunity for accessing the network is assigned to each of the SUs. Non-supported SUs with lower network access opportunity are sequentially removed until the remaining SUs along with all PUs reach a feasible power vector. In [2], assuming the same QoS (i.e., target-SINR) for all the SUs, an algorithm is proposed in which the SUs are sorted according to their link gain ratios (i.e., the ratio of the link gain of the SUs toward secondary receiving point and the corresponding link gain toward primary receiving point) and non-supported SUs are removed by using the bisection search algorithm.

The removal criterion proposed in [2], [3], [8] and [9] requires a centralized node to know all the system parameters including instantaneous channel state information (CSI) between all nodes, the target-SINR and maximum transmit power levels for all users. This causes heavy signalling overheads. Furthermore, the complexity of removal algorithms proposed in [2], [3], [8], [9] are of $O(|\mathcal{U}^{\mathrm{s}}|^4)$, $O(|\mathcal{U}^{\mathrm{s}}|^3)$, $O(|\mathcal{U}^{\mathrm{s}}|^3)$, $O(|\mathcal{U}^{\mathrm{s}}|^2 \log |\mathcal{U}^{\mathrm{s}}|)$, respectively, where $|\mathcal{U}^{\mathrm{s}}|$ is the number of SUs (refer to Table I).

In [10], a distributed prioritized power control algorithm is proposed in which the feasibility of target-SINRs for SUs under the constraint of zero-outage for PUs is individually checked by each SU in a distributed manner, where an SU removes itself if that user is unable to reach its target-SINR and/or its

TABLE I
SUMMARY OF RELATED WORK AND COMPARISON WITH OUR PROPOSED APPROACH

Ref.	Type of the Cognitive Network	Type of the Primary Network	Impact of PUs' Transmissions on SUs' Receivers	Solution Category	Assumptions and Required Information
[8]	Multiple secondary transmitter/receiver pairs	Single primary receiver	No	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^4)$	Full knowledge
[9]	Multiple secondary transmitter/receiver pairs	Multiple primary TV receivers and single primary TV station	Yes	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^3)$	Full knowledge
[3]	Multiple secondary transmitter/receiver pairs	Multiple primary transmitter/receiver pairs	No	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^3)$	Full knowledge
[2]	Multiple secondary transmitter/receiver pairs	Multiple primary transmitter/receiver pairs	No	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^2 \log \lvert\mathcal{U}^{\mathrm{s}}\rvert)$	Full knowledge
[11]	Single-cell served by a secondary BS with antenna array	Single primary receiver and single primary transmitter	Yes	Distributed	Unconstrained PU's transmit power
[6]	Multiple secondary transmitter/receiver pairs	Single primary receiver and multiple primary transmitters	Yes	Distributed	Iterative signal exchanging between SUs to schedule the round robin turns of power updates, updating the current activated and deactivated SUs, broadcasting a warning signals to SUs by primary receiver if the interference temperature constraint limit is violated
[4]	Multiple secondary transmitter/receiver pairs	Single primary receiver	No	Distributed	Each SU is aware of the target-SINR of other users, the path-gain from other interfering users toward its receiver and from that user to the receiver of the other users, iterative message passing between SUs to exchange their instantaneous transmit power levels and dual variables
[10]	Single-cell	Single primary receiver	Yes	Distributed	PUs and SUs connect to a single BS, each SU is aware of total received power temperature and noise level at the BS
Our work	Multi-cell cognitive network	Multi-cell primary network	Yes	Distributed	Iterative broadcasting the ratio of total received power to total received power temperature at the primary receivers to SUs

existence causes outage of a high-priority user. However, [10] focuses on single-cell networks where all SUs and PUs are served by a single base-station. Our current paper considers a sufficiently general system model of multi-cellular networks where a multi-cell secondary radio network coexists with a multi-cell primary network. In [11], a distributed algorithm is introduced to minimize the total transmit power of primary and secondary links using antenna arrays. However, the PUs are allowed to increase their transmit power levels without bounds, which is not practical. In [6], a power and admission control algorithm is proposed to maximize the aggregate throughput for the maximum number of SUs that can be admitted to the network under the constraint of PUs' interference temperature limit. However, this algorithm incurs a significant amount of computation and signalling overhead.

Different from the existing work in the literature, in this paper, we design distributed uplink power control algorithms with reduced signalling overhead and computation complexity for a general system model where there exist multiple primary and secondary transmitter/receivers in a multi-cell environment. The objective is to support the maximal number of SUs with their target-SINRs subject to the constraint of zero-outage ratio for the PUs. In contrast to the works in [2]–[4] where transmit power of primary networks and thus the interference caused by primary networks on secondary networks are assumed fixed, the dynamics of PUs' transmit power is considered in our system model. Furthermore, in contrast to [4], [6], [10], and [11], which consider a CRN with only a single primary receiver, we consider a multi-cell CRN coexisting with a multi-cell primary network as is the case in practice. *Note that, existence of multiple receivers or primary base-stations (BSs) requires us to satisfy the corresponding interference temperature at each BS which makes the problem of power control in underlay CRNs more challenging*. This is due to the fact that, for controlling the transmit power of a given SU, the amount of interference imposed by that SU at different points of primary receivers has to be taken into account.

III. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Model and Notations

Consider an underlay interference-limited cognitive wireless network where a secondary cellular network coexists with a primary cellular network. The secondary network consists of a set of SUs denoted by $\mathcal{U}^{\mathrm{s}}$ which are served by a set of secondary base-stations (SBSs) denoted by $\mathcal{B}^{\mathrm{s}}$. The primary network consists of a set of primary base-stations (PBSs) denoted by $\mathcal{B}^{\mathrm{p}}$ serving the set of PUs denoted by $\mathcal{U}^{\mathrm{p}}$. We assume a fixed base-station assignment in both primary and secondary networks, i.e., each PU or SU is already associated with a fixed BS in the corresponding cells. Let us denote the set of PUs associated to BS $k \in \mathcal{B}^{\mathrm{p}}$ by $\mathcal{U}_k^{\mathrm{p}}$ and the set of SUs associated to BS $k \in \mathcal{B}^{\mathrm{s}}$ by $\mathcal{U}_k^{\mathrm{s}}$. Thus we have $\mathcal{U}^{\mathrm{p}} = \cup_{k\in\mathcal{B}^{\mathrm{p}}}\mathcal{U}_k^{\mathrm{p}}$ and $\mathcal{U}^{\mathrm{s}} = \cup_{k\in\mathcal{B}^{\mathrm{s}}}\mathcal{U}_k^{\mathrm{s}}$. Let us also denote the set of all users by $\mathcal{U} = \mathcal{U}^{\mathrm{p}} \cup \mathcal{U}^{\mathrm{s}}$ and the set of all base stations by $\mathcal{B} = \mathcal{B}^{\mathrm{p}} \cup \mathcal{B}^{\mathrm{s}}$.

Let p_i be the transmit power of user i and $0 \le p_i \le \overline{p}_i$, where $\overline{p}_i$ is the upper limit of the transmit power for user i. Let $\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}$ imply $0 \le p_i \le \overline{p}_i$ for all $i \in \mathcal{U}$. The BS assigned to user i is denoted by $b_i \in \mathcal{B}$ and the path gain from user j to the BS b_i is denoted by $h_{b_i,j}$, and thus the received power of user j at the BS assigned to user i is $p_j h_{b_i,j}$. Noise power at each receiver is assumed to be additive white Gaussian.

The receiver is assumed to be a conventional matched filter. Thus, for a given transmit power vector $\mathbf{p}$, the SINR of user i achieved at its receiver, denoted by γ_i is

$$\gamma_i(\mathbf{p}) \triangleq \frac{p_i h_{b_i,i}}{\sum\limits_{j\in\mathcal{U}, j\neq i} p_j h_{b_i,j} + \sigma_{b_i}^2}, \tag{1}$$

where $\sigma_{b_i}^2$ is the noise power at the receiver of user i. An SINR vector is denoted by $\gamma = [\gamma^{\mathrm{p}}, \gamma^{\mathrm{s}}]$, where γ^{p} and γ^{s} are SINRs of PUs and SUs, respectively.

Let us denote the total received power at a given base station $k \in \mathcal{B}$ by

$$\varphi_k(\mathbf{p}) = \sum_{i\in\mathcal{U}} p_i h_{k,i} + \sigma_k^2. \tag{2}$$

The effective interference for user i is denoted by R_i, and is defined as the ratio of interference caused to each user i to the path gain to its assigned BS, that is

$$R_i(\mathbf{p}) \triangleq \frac{I_i(\mathbf{p})}{h_{b_i,i}}, \tag{3}$$

where $I_i(\mathbf{p}) = \sum\limits_{j\neq i} p_j h_{b_i,j} + \sigma_{b_i}^2$ is the total interference caused to user i at its receiver. Let us also define the effective SINR of user i by

$$\theta_i(\mathbf{p}) = \frac{\gamma_i(\mathbf{p})}{\gamma_i(\mathbf{p}) + 1}, \tag{4}$$

which is the ratio of received power of user i to the total received power plus noise, i.e., $\theta_i(\mathbf{p}) = \frac{p_i h_{b_i,i}}{\varphi_{b_i}(\mathbf{p})}$.

The target-SINR of each user i is denoted by $\widehat{\gamma}_i$, and is usually equivalent to a maximum tolerable bit error rate (BER) below which the user is not satisfied. Correspondingly, the target-effective SINR is $\widehat{\theta}_i = \frac{\widehat{\gamma}_i}{\widehat{\gamma}_i+1}$. Given a transmit power vector, user i is supported if $\gamma_i(\mathbf{p}) \ge \widehat{\gamma}_i$, or equivalently, if $\theta_i(\mathbf{p}) \ge \widehat{\theta}_i$. Given a transmit power vector $\mathbf{p}$, let us denote the set of supported users by $\mathcal{S}(\mathbf{p}) = \{i \in \mathcal{U} | \theta_i(\mathbf{p}) \ge \widehat{\theta}_i\}$. We also denote the set of supported SUs and PUs by $\mathcal{S}^{\mathrm{p}}(\mathbf{p}) = \mathcal{S}(\mathbf{p}) \cap \mathcal{U}^{\mathrm{p}}$ and $\mathcal{S}^{\mathrm{s}}(\mathbf{p}) = \mathcal{S}(\mathbf{p}) \cap \mathcal{U}^{\mathrm{s}}$, respectively. Their complementary sets are $\mathcal{S}'(\mathbf{p}) = \mathcal{U} - \mathcal{S}(\mathbf{p})$, $\mathcal{S}'^{\mathrm{p}}(\mathbf{p}) = \mathcal{U}^{\mathrm{p}} - \mathcal{S}^{\mathrm{p}}(\mathbf{p})$, and $\mathcal{S}'^{\mathrm{s}}(\mathbf{p}) = \mathcal{U}^{\mathrm{s}} - \mathcal{S}^{\mathrm{s}}(\mathbf{p})$. The cardinality of a given set $\mathcal{A}$ is denoted by $|\mathcal{A}|$. Given a transmit power vector $\mathbf{p}$, let us define the outage-ratio for primary and secondary users denoted by $O^{\mathrm{p}}(\mathbf{p})$ and $O^{\mathrm{s}}(\mathbf{p})$, respectively, as follows:

$$O^{\mathrm{p}}(\mathbf{p}) = \frac{|\mathcal{S}'^{\mathrm{p}}(\mathbf{p})|}{|\mathcal{U}^{\mathrm{p}}|} \quad \text{and} \quad O^{\mathrm{s}}(\mathbf{p}) = \frac{|\mathcal{S}'^{\mathrm{s}}(\mathbf{p})|}{|\mathcal{U}^{\mathrm{s}}|}. \tag{5}$$

In the TPC method proposed in [7], [13], the transmit power for each user i is iteratively set by using

$$p_i(t+1) = \min\left\{\overline{p}_i, f_i^{(\mathrm{T})}(\mathbf{p}(t))\right\}, \tag{6}$$

where

$$f_i^{(\mathrm{T})}(\mathbf{p}(t)) = \widehat{\gamma}_i R_i(\mathbf{p}(t)) \tag{7}$$

in which $R_i(t)$ is the effective interference caused to user i at iteration t and $\overline{p}_i$ is the maximum transmit power constraint. When $p_i(t) \neq 0$, the power-update function in TPC can be rewritten as: $f_i^{(\mathrm{T})}(\mathbf{p}(t)) = \widehat{\theta}_i \varphi(\mathbf{p}(t)) = \frac{\widehat{\theta}_i}{\theta_i(t)} p_i(t) = \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)$, where $\gamma_i(\mathbf{p}(t))$ and $\theta_i(t)$ are the actual SINR and the effective SINR of user i at iteration t, respectively. Convergence to a unique fixed-point[1] is guaranteed for the TPC in both feasible and infeasible systems. However, it suffers from a severe drawback in infeasible systems. Since users employing the TPC rigidly track their target-SINRs, there always exist a few users transmitting at their maximum power without obtaining their target-SINRs, which results in high outage-ratio and high power consumption.

B. Problem Statement

Using matrix notations, the relation between the transmit power vector and the SINR vector can be rewritten as

$$\mathbf{p} = \mathbf{G}.\mathbf{p} + \eta, \tag{8}$$

where the (i,j) component of $\mathbf{G}$ is $G_{i,j} = \frac{h_{b_i,j}\gamma_i}{h_{b_i,i}}$ if $i \neq j$, and $G_{i,j} = 0$ if $i = j$, and the i-th component of η is $\eta_i = \frac{\sigma_{b_i}^2 \gamma_i}{h_{b_i,i}}$.

Definition 1: The target-SINRs of users in a given subset $\mathcal{A} \subseteq \mathcal{U}$ are feasible if there exists a power vector $\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}$ that satisfies the target-SINRs of users in $\mathcal{A}$. In addition, the *system* is *feasible* if the target-SINR vector for all users (i.e., when $\mathcal{A} = \mathcal{U}$) is feasible, otherwise the system is *infeasible*.

It is shown in [5] that the necessary condition for the feasibility of a given SINR vector γ is $\rho(\mathbf{G}) < 1$, where $\rho(\mathbf{G})$ is the spectral radius (maximum eigenvalue) of matrix $\mathbf{G}$. This would be a sufficient condition only if there is no upper limit on transmit power of users (i.e., $\overline{p}_i = \infty$).

[1]In a distributed power control algorithm, each user i updates its transmit power by a power-update function $f_i(\mathbf{p})$, that is, $p_i(t+1) = f_i(\mathbf{p}(t))$, where $\mathbf{p}(t)$ is the transmit power vector at time t. The fixed-point of the power update function, denoted by $\mathbf{p}^*$, is obtained by solving $\mathbf{p}^* = \mathbf{f}(\mathbf{p}^*)$.

Throughout this paper, we suppose that the target-SINRs for the PUs are feasible, i.e., there exists a transmit power vector $\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}$ for which $O^{\mathrm{p}}(\mathbf{p}) = 0$. But the target-SINRs for all PUs and SUs together may be infeasible. In an infeasible system, the minimal number of SUs should be gradually removed subject to the constraint that all the PUs are supported with their target-SINRs (zero-outage-ratio for the PUs). We define this as the problem of minimizing the outage-ratio of SUs subject to zero-outage-ratio of PUs as follows:

$$\min_{\mathbf{0}\le\mathbf{p}\le\overline{\mathbf{p}}} O^{\mathrm{s}}(\mathbf{p}) \qquad \text{subject to } O^{\mathrm{p}}(\mathbf{p}) = 0, \tag{9}$$

in which the constraint $O^{\mathrm{p}}(\mathbf{p}) = 0$ means $\mathcal{S}^{\mathrm{p}}(\mathbf{p}) = \mathcal{U}^{\mathrm{p}}$, i.e., $\gamma_i(\mathbf{p}) \ge \widehat{\gamma}_i$, $\forall\, i \in \mathcal{U}^{\mathrm{p}}$, which is assumed to be feasible.

IV. Proposed Distributed Power Control Algorithm

In this section we present our proposed distributed power control algorithm for uplink power control in CRNs. To avoid outage of a PU due to the existence of SUs, a new upper-limit constraint is imposed on the transmit power levels of SUs in addition to their maximum transmit power constraint $\overline{p}_i$, so that the total interference caused by the SUs to the PUs is kept below a given threshold.

A. Total Received-Power-Temperature at the Primary Base Stations (PBSs)

To guarantee a zero-outage ratio for PUs, the total received power plus noise at each PBS must be below a given threshold, as explained and obtained below. Given the total received power plus noise at the PBS $k \in \mathcal{B}^{\mathrm{p}}$, i.e., $\varphi_k(\mathbf{p})$, the effective target-SINR of user $i \in \mathcal{U}^{\mathrm{p}}$ is reachable if and only if $0 \le \frac{\widehat{\theta}_i}{h_{b_i,i}}\varphi_{b_i}(\mathbf{p}) \le \overline{p}_i$. Let $\overline{\varphi}_k$ denote the maximum value of the total received power plus noise at the BS $k \in \mathcal{B}^{\mathrm{p}}$ that can be tolerated by all of its associated PUs. We refer $\overline{\varphi}_k$ as the *total received-power-temperature* for PBS k, which is formally defined and obtained as follows:

$$\overline{\varphi}_k = \max\left\{\varphi \,|\, 0 \le \frac{\widehat{\theta}_i}{h_{k,i}}\varphi \le \overline{p}_i,\ \forall\, i \in \mathcal{U}_k^{\mathrm{p}}\right\} = \min_{i\in\mathcal{U}_k^{\mathrm{p}}}\left\{\frac{\overline{p}_i h_{k,i}}{\widehat{\theta}_i}\right\}. \tag{10}$$

Lemma 1: If the transmit power vector $\mathbf{p}$ satisfies the SINR requirements of all PUs then we have $\varphi_k(\mathbf{p}) \le \overline{\varphi}_k$, for all $k \in \mathcal{B}^{\mathrm{p}}$, or equivalently, $\max_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\varphi_k(\mathbf{p})}{\overline{\varphi}_k}\right\} \le 1$.

As can be seen, the total received-power-temperature for each PBS $k \in \mathcal{B}^{\mathrm{p}}$, i.e., $\overline{\varphi}_k$ is a dynamic function of noise level, target-SINRs, channel gains, and maximum transmit power levels for users associated to PBS k. In fact, the values of $\overline{\varphi}_k$ for all $k \in \mathcal{B}^{\mathrm{p}}$ indicate the amount of interference tolerability of PBSs at the primary network in the underlay spectrum access strategy. The value of total received-power-temperature for each PBS is dynamically decreased (increased) as the number of its associated PUs are increased (decreased) and/or channel status of primary network becomes weaker (stronger).

Note that the total received-power-temperature $\overline{\varphi}_k$ is obtained by each PBS based on information pertinent to its own associated users only. Thus each PBS $k \in \mathcal{B}^{\mathrm{p}}$ can compute the value of $\overline{\varphi}_k$ in a distributed manner without requiring to know the channel information of other PUs associated to other base-stations $l \in \mathcal{B}^{\mathrm{p}}$. *If, instead of putting a constraint on total received power, we put a constraint on interference caused by the SUs to the PBS (i.e., the so called interference temperature limit in the literature), then the interference temperature limit for each PBS would depend on the channel gains and the target-SINRs of all PUs including those PUs not associated to that PBS as explained below.*

The maximum value of the total interference caused by the SUs to the BS $k \in \mathcal{B}^{\mathrm{p}}$ that can be tolerated by all of its associated PUs, denoted by $\overline{I}_k$, is formally defined and obtained as follows:

$$\begin{aligned}\overline{I}_k &= \max\left\{I^{\mathrm{s}} \,|\, 0 \le \frac{\widehat{\theta}_i}{h_{k,i}}\left(I_k^{\mathrm{intp}} + I_k^{\mathrm{extp}} + I^{\mathrm{s}} + \sigma_k^2\right) \le \overline{p}_i,\ \forall\, i \in \mathcal{U}_k^{\mathrm{p}}\right\}\\ &= \min_{i\in\mathcal{U}_k^{\mathrm{p}}}\left\{\frac{\overline{p}_i h_{k,i}}{\widehat{\theta}_i}\right\} - \left(I_k^{\mathrm{intp}} + I_k^{\mathrm{extp}} + \sigma_k^2\right),\end{aligned} \tag{11}$$

where I_k^{intp} is the total intra-cell interference (total received power by PUs associated to PBS k) and I_k^{extp} is the total (primary inter-cell) interference caused by those PUs not associated to PBS k. As can be seen, the interference temperature at each PBS k (i.e., $\overline{I}_k$) not only depends on the channel gains and the target-SINR requirements for the associated PUs, but also on the instantaneous value of the total interference caused by those PUs not associated to the PBS k. This is in contrast to the total received-power-temperature which depends on the channel status and the target-SINR requirements of its associated PUs only. For this reason, unlike the traditional literature, we focus on the total received-power-temperature limit as a constraint imposed on the transmit power levels of the SUs. This approach enables us to address the problem of distributed uplink power control in cellular CRNs as will be demonstrated in the following sections.

B. Proposed Distributed Power Control Algorithm

Our proposed TPC with PU-protection algorithm (TPC-PP), as summarized in **Algorithm 1** has the following distributed power-update function:

$$p_i(t+1) = \begin{cases}\min\left\{\overline{p}_i,\ \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}p_i(t)\right\}, & \text{for all } i \in \mathcal{U}^{\mathrm{p}}\\ \min\left\{\overline{p}_i,\ \beta(t)p_i(t),\ \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}p_i(t)\right\}, & \text{for all } i \in \mathcal{U}^{\mathrm{s}},\end{cases} \tag{12}$$

where $\beta(t) = \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$.

Algorithm 1 TPC with PU-protection (TPC-PP)

Set $t := 1$, for each user $i \in \mathcal{U}$, initialize the transmit power randomly $p_i(t) = \dot{p}_i$ where $\dot{p}_i \in (0, \bar{p}_i]$ and estimate the CSI values from previous time slot.
repeat
  for all PU $i \in \mathcal{U}^{\mathrm{p}}$ **do**
    Obtain the parameter $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}$ from its own PBS.
    Update the power as

$$p_i(t+1) := \min\left\{\bar{p}_i, \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}.$$

  end for
  Each PBS $k \in \mathcal{B}^{\mathrm{p}}$ multicast the parameter $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ to all SU $i \in \mathcal{U}^{\mathrm{s}}$.
  for each SU $i \in \mathcal{U}^{\mathrm{s}}$ **do**
    Obtain the parameter $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}$ from its own SBS.
    Find $\beta(t) := \min_{k \in \mathcal{B}^{\mathrm{p}}} \left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$.
    Update the power as

$$p_i(t+1) := \min\left\{\bar{p}_i, \beta(t) p_i(t), \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}.$$

  end for
  Update the power vector $\mathbf{p}(t+1) := [p_i(t+1)]_{\forall i \in \mathcal{U}}$.
  Update $t := t+1$.
until $t = T_{\max}$ **or** convergence to any fixed point.

In TPC-PP, each PU employs the TPC. However, each SU employs the TPC as long as the total received power plus noise power at each PBS k, i.e., $\varphi_k(t)$ is less than the corresponding total received-power-temperature $\overline{\varphi}_k$, otherwise the SU updates its transmit power proportional to $\frac{\overline{\varphi}_k}{\varphi_k(t)} p_i(t)$, which is equivalent to setting the transmit power $p_i(t+1)$ to $\min_{k \in \mathcal{B}^{\mathrm{p}}} \left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\} p_i(t)$. The TPC algorithm is indeed the same as the closed-loop power control algorithm, since the ratio of $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)$ in the TPC algorithm can be viewed as the commands of increasing or decreasing the power in closed-loop power control algorithm, corresponding to $\gamma_i(\mathbf{p}(t)) < \widehat{\gamma}_i$ and $\gamma_i(\mathbf{p}(t)) > \widehat{\gamma}_i$, respectively. Similarly, the term $\frac{\overline{\varphi}_k}{\varphi_k(t)} p_i(t)$ can also be viewed as a power-updating command issued by the PBS to SUs. The proposed power control algorithm for the SUs can be interpreted as follows. Each SU receives two power-updating commands at each iteration, one is unicast from its own receiver, in terms of $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}$, and the other ones are multicast from each PBS to all SUs, in terms of $\frac{\overline{\varphi}_k}{\varphi_k(t)}$.

Indeed, the TPC-PP algorithm uses a mixed-strategy for spectrum access as explained in the following. When there are many PUs with large target-SINR requirements associated to a PBS and/or the corresponding channel gains are poor, the total received-power-temperature for that PBS is set to a very small value [according to (10)]. This corresponds to spectrum overlay strategy. On the other hand, when the number and/or the target-SINR requirements of the PUs actively associated to each PBS is moderate and/or the channel gains are good, the values of total received-power-temperature for the PBSs can be non-zero. These values would indicate the amount of interference tolerability of the primary network in the spectrum underlay strategy. Therefore, by dynamically setting the value of the total received-power-temperature for each PBS in an optimum manner using (10), a mixed-strategy is adopted.

V. Analysis of the Proposed Algorithm

A. Signalling Overhead

In our proposed algorithm, in addition to information that each user requires to update its transmit power using the TPC at each iteration, each SU needs to know the ratio of the total received-power-temperature to the instantaneous total received power plus noise for each PBS, i.e., $\frac{\overline{\varphi}_k}{\varphi_k(t)}$, which is provided by the primary base stations. Thus, in comparison with TPC, the additional signalling overhead that TPC-PP incurs is that it requires each PBS k to iteratively provide the SUs with the value of $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ (via a broadcast message in the control channel). Each PBS k may broadcast the values of $\overline{\varphi}_k$ and $\varphi_k(t)$, individually, or the ratios, i.e., $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ to the SUs. Note that the value of $\overline{\varphi}_k$ needs to be updated by PBS k only when one of its associated PUs, who has the minimum value of $\frac{\bar{p}_i h_{k,i}}{\widehat{\theta}_i}$ among all associated PUs, leaves or enters the system. However, in contrast, the value of $\varphi_k(t)$ needs to be updated at each iteration. Since in practice each SU may cause severe interference only to its nearby PBS, each PBS should inform the values of $\overline{\varphi}_k$ and $\varphi_k(t)$ only to its nearby SUs. Alternatively, each SBS can collect the values of $\overline{\varphi}_k$ and $\varphi_k(t)$ from all the nearby PBSs, and feedback only its minimum ratio, i.e., $\min_{k \in \mathcal{B}^{\mathrm{p}}} \left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$ to its associated SUs. In a practical implementation, the feedback information can be quantized and theses quantized feedback information (bits) can be multicast. This is similar to CSI quantization and feedback commonly used in practice. With this implementation, we can control the feedback overhead and performance trade-off by choosing appropriate number of bits for feedback. These feedback information can also be sent to SBSs by PBSs via a possible wired network between them and then SBSs send these feedback to their own SUs.

One may replace $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ with $\frac{\bar{I}_k}{I_k^{\mathrm{s}}(t)}$ in TPC-PP (12), where $\bar{I}_k$ is the interference temperature given by (11), and $I_k^{\mathrm{s}}(t)$ is the instantaneous value of interference caused by all SUs to the PBS k. In this case, all of the analytical results developed in following sections are still valid. However, note that, the former is preferred to the latter from a practical point of view. This is because in the latter case, in addition to $I_k^{\mathrm{s}}(t)$, each PBS k needs to know the value of $\bar{I}_k$, which is a function of the instantaneous value of the total interference caused by all of those PUs not associated to PBS k, as explained in Section IV-A. Thus, given an instantaneous value of the total received power at the PBS, each PBS requires to compute the total interference caused by its associated PUs and all of non-associated PUs separately. On the other hand, to calculate $\frac{\overline{\varphi}_k}{\varphi_k(t)}$, PBS k can easily obtain its total received-power-temperature by using (10) and the information pertinent to its associated PUs, and also can easily measure the instantaneous value of the total received power at its receiver without requiring to know the individual instantaneous values of interference caused to it by all the PUs (i.e., both the associated and non-associated ones).

B. Existence of Fixed-Point and Its Properties

In this section, we show that there exists at least one fixed-point for our proposed power-update function and all of its fixed-points guarantee zero-outage for the PUs. For a given a target-SINR vector $\gamma=[\gamma^{\mathrm{p}},\gamma^{\mathrm{s}}]$, let $\mathbf{p}^{*\mathrm{T}}(\gamma)$ denote the fixed-point of the TPC power-update function, i.e., $p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\gamma_i R_i(\mathbf{p}^{*\mathrm{T}})\right\}$ for all $i\in\mathcal{U}$, supposing that all PUs and SUs employ the TPC with the target-SINR vector of γ.

Lemma 2: Given a target-SINR vector $\widehat{\gamma}=[\widehat{\gamma}^{\mathrm{p}},\widehat{\gamma}^{\mathrm{s}}]$, the corresponding fixed-point of the TPC power-update function $\mathbf{p}^{*\mathrm{T}}(\gamma)$, and corresponding total received-power-temperature $\overline{\varphi}_k$ for each PBS $k\in\mathcal{B}^{\mathrm{p}}$ obtained from (10), the following observations can be made:

(a) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\le 1$ (or equivalently, if there exists at least one PBS $k\in\mathcal{B}^{\mathrm{p}}$ for which $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))\ge\overline{\varphi}_k$, which implies that there may exist one PU who is in outage due to TPC), then there exists at least one transmit power vector $\mathbf{p}$ for which the following equalities and inequalities hold:

$$\begin{aligned}&\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p})}\right\}=1\\&0\le p_i\le\overline{p}_i,\quad\text{for all } i\in\mathcal{U}\\&\gamma_i(\mathbf{p})=\widehat{\gamma}_i^{\mathrm{p}},\quad\text{for all } i\in\mathcal{U}^{\mathrm{p}}\\&\gamma_i(\mathbf{p})\le\widehat{\gamma}_i^{\mathrm{s}},\quad\text{for all } i\in\mathcal{U}^{\mathrm{s}}.\end{aligned}\tag{13}$$

(b) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$ (or equivalently, if $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_k$ for all PBS $k\in\mathcal{B}^{\mathrm{p}}$, which implies zero-outage for PUs by TPC), then no transmit power vector exists which satisfies all of the conditions above.

Proof:

(a) Let $l=\arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}$. The target-SINRs of PUs are feasible, i.e., $\gamma'=[\widehat{\gamma}^{\mathrm{p}},0]$ is feasible and thus it is achievable by the TPC and from **Lemma 1** we conclude that $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\gamma'))\le\overline{\varphi}_k$ for all $k\in\mathcal{B}^{\mathrm{p}}$. Since $\gamma'\le\widehat{\gamma}$, we have $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\gamma'))\le\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$ for all $k\in\mathcal{B}^{\mathrm{p}}$. Therefore, if $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\le 1$, or equivalently, if $\overline{\varphi}_l\le\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$, we have $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma'))\le\overline{\varphi}_l\le\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$. From this and by noting that $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma))$ is a continuous function of γ (because the functions $\varphi_l(\mathbf{p})$ and $\mathbf{p}^{*\mathrm{T}}(\gamma)$ are continuous), we conclude from the Intermediate-Value Theorem [14] that there exists at least one SINR vector γ where $\gamma'\le\gamma\le\widehat{\gamma}$ for which $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma))=\overline{\varphi}_l$. Thus there exists a transmit power vector $\mathbf{p}=\mathbf{p}^{*\mathrm{T}}(\gamma)$ that satisfies (13) (because $\varphi_l(\mathbf{p})=\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma))=\overline{\varphi}_l$ and $\gamma'\le\gamma\le\widehat{\gamma}$ corresponds to two last constraints of (13)).

(b) Let $l=\arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}$. If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, then we have $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_k$ for all $k\in\mathcal{B}^{\mathrm{p}}$, and thus $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_l$. When $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_l$, if there exists a transmit power vector $\mathbf{p}$ that satisfies (13), we have $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\varphi_l(\mathbf{p})=\overline{\varphi}_l$. From this we conclude that $\gamma_i(\mathbf{p})>\widehat{\gamma}_i$ holds for at least a user $i\in\mathcal{U}$. Because, otherwise, we have $\gamma_i(\mathbf{p})\le\widehat{\gamma}_i$ for all $i\in\mathcal{U}$ (i.e., $\gamma(\mathbf{p})\le\widehat{\gamma}$) and hence $\varphi_l(\mathbf{p})\le\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$. Since one can show that for any two feasible SINRs, γ_1 and γ_2 and their corresponding power vector $\mathbf{p}_1$ and $\mathbf{p}_2$, if $\gamma_1\le\gamma_2$ then $\mathbf{p}_1\le\mathbf{p}_2$, and thus $\varphi(\mathbf{p}_1)\le\varphi(\mathbf{p}_2)$. This contradicts $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\varphi_l(\mathbf{p})=\overline{\varphi}_l$. This implies that when $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, no transmit power vector $\mathbf{p}$ exists that satisfies (13). ■

Theorem 1: Similar to **Lemma 2**, let $\mathbf{p}^{*\mathrm{T}}$ be the fixed-point of the TPC power-update function when all users employ the TPC algorithm.

(a) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\le 1$, then any transmit power vector which satisfies the conditions in (13) is a fixed-point of TPC-PP. In this case, the fixed-point of the TPC-PP is not generally unique.

(b) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, then the fixed-point of TPC-PP is unique and the same as $\mathbf{p}^{*\mathrm{T}}$.

Proof: Let $l=\arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}$. We consider the following two cases.

Case (a): From **Lemma 2** we know that, if $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\le 1$, then there exists a transmit power vector $\widetilde{\mathbf{p}}$ that satisfies the conditions in (13). To prove that $\widetilde{\mathbf{p}}$ is the fixed-point of TPC-PP, we need to show the following:

$$\widetilde{p}_i=\min\left\{\overline{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{p}}\tag{14}$$

$$\widetilde{p}_i=\min\left\{\overline{p}_i,\frac{\overline{\varphi}_l}{\varphi_l(\widetilde{\mathbf{p}})}\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{s}}.\tag{15}$$

From (13) we conclude that $\widetilde{p}_i=\gamma_i(\widetilde{\mathbf{p}})R_i(\widetilde{\mathbf{p}})=\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})$ and $\widetilde{p}_i\le\overline{p}_i$ for all $i\in\mathcal{U}^{\mathrm{p}}$ and thus (14) holds. Also, since $\varphi_l(\widetilde{\mathbf{p}})=\overline{\varphi}_l$, we have $\min\{\overline{p}_i,\frac{\overline{\varphi}_l}{\varphi_l(\widetilde{\mathbf{p}})}\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\}=\min\{\overline{p}_i,\widetilde{p}_i,\widehat{\gamma}_i R_i(\mathbf{p})\}$ for all $i\in\mathcal{U}^{\mathrm{s}}$. Hence to prove (15), we only need to show that $\widetilde{p}_i=\min\{\overline{p}_i,\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\}$ holds for all $i\in\mathcal{U}^{\mathrm{s}}$. From (13) we know that $\widetilde{p}_i\le\overline{p}_i$ and $\widetilde{p}_i=\gamma_i(\widetilde{\mathbf{p}})R_i(\widetilde{\mathbf{p}})\le\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})$, and consequently, $\widetilde{p}_i=\min\{\overline{p}_i,\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\}$ holds for all $i\in\mathcal{U}^{\mathrm{s}}$, and hence (15) holds. This completes the proof.

Case (b): If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, we first show that $\mathbf{p}^{*\mathrm{T}}$ is a fixed-point of TPC-PP and then show that this fixed-point is unique. To show the former, we need to show that

$$p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\widehat{\gamma}_i R_i\left(\mathbf{p}^{*\mathrm{T}}\right)\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{p}}\tag{16}$$

$$p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\frac{\overline{\varphi}_l}{\varphi_l\left(\mathbf{p}^{*\mathrm{T}}\right)}p_i^{*\mathrm{T}},\widehat{\gamma}_i R_i\left(\mathbf{p}^{*\mathrm{T}}\right)\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{s}}.\tag{17}$$

Since $\mathbf{p}^{*\mathrm{T}}$ is the fixed-point of the TPC, we have $p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\widehat{\gamma}_i R_i(\mathbf{p}^{*\mathrm{T}})\right\}$ for all $i\in\mathcal{U}$ and thus (16) holds. From

$\varphi_l(\mathbf{p}^{*\mathrm{T}}) < \overline{\varphi}_l$, we conclude that $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p})} p_i^{*\mathrm{T}} > p_i^{*\mathrm{T}}$. From this and from $p_i^{*\mathrm{T}} = \min\left\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^{*\mathrm{T}})\right\}$ for all $i \in \mathcal{U}$, (17) is concluded and thus the proof is completed. ■

Theorem 2: Given a fixed-point $\mathbf{p}^*$ for the power-update function of our proposed algorithm, we have $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} \geq 1$ (or equivalently, $\varphi_k(\mathbf{p}^*) \leq \overline{\varphi}_k$ for all $k \in \mathcal{B}^{\mathrm{p}}$). Furthermore,

(a) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} = 1$, then $\mathbf{p}^*$ satisfies the conditions in (13). In this case, the fixed-point for the power-update function of the TPC-PP is generally not unique.

(b) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} > 1$, then the fixed-point $\mathbf{p}^*$ is the same as the fixed-point of the TPC. In this case, the fixed-point for the power-update function of the TPC-PP is unique.

Proof: If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} < 1$, then we have $p_i^* > \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}{\overline{\varphi}_k}\right\} p_i^*$, and thus the fixed-point constraint (15) cannot hold. Therefore, for any fixed-point we have $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} \geq 1$. We now prove parts (a) and (b).

Part (a): If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} = 1$, we have $\widehat{\gamma}_i R_i(\mathbf{p}^*) = \frac{\widehat{\theta}_i}{h_{b_i,i}} \varphi_{b_i}(\mathbf{p}^*) \leq \frac{\widehat{\theta}_i}{h_{b_i,i}} \overline{\varphi}_{b_i} \leq \overline{p}_i$, for all $i \in \mathcal{U}^{\mathrm{p}}$, in which the last inequality holds because $\overline{\varphi}_{b_i} = \min_{j\in\mathcal{U}^{\mathrm{p}}_{b_i}}\left\{\frac{\overline{p}_j h_{b_i,j}}{\widehat{\theta}_j}\right\} \leq \frac{\overline{p}_i h_{b_i,i}}{\widehat{\theta}_i}$, for all $i \in \mathcal{U}^{\mathrm{p}}$. Thus $p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\} = \widehat{\gamma}_i R_i(\mathbf{p}^*)$, for all $i \in \mathcal{U}^{\mathrm{p}}$ which implies that $\gamma_i(\mathbf{p}^*) = \widehat{\gamma}_i$ for all $i \in \mathcal{U}^{\mathrm{p}}$. In addition, from $p_i^* = \min\left\{\overline{p}_i, \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^*, \widehat{\gamma}_i R_i(\mathbf{p}^*)\right\}$ for all $i \in \mathcal{U}^{\mathrm{s}}$, we conclude $p_i^* \leq \widehat{\gamma}_i R_i(\mathbf{p}^*)$, or equivalently, $\gamma_i(\mathbf{p}^*) \leq \widehat{\gamma}_i$ for all $i \in \mathcal{U}^{\mathrm{s}}$. Thus $\mathbf{p}^*$ satisfies the constraints in (13).

Part (b): Since $\mathbf{p}^*$ is a fixed-point for our proposed power update function, it satisfies the following fixed-point constraints:

$$p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\}, \text{ for all } i \in \mathcal{U}^{\mathrm{p}}, \tag{18}$$

$$p_i = \min\left\{\overline{p}_i, \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^*, \widehat{\gamma}_i R_i(\mathbf{p}^*)\right\}, \text{ for all } i\in\mathcal{U}^{\mathrm{s}}. \tag{19}$$

To show that $\mathbf{p}^*$ is a fixed-point of the TPC, we need to show that $p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\}$ for all $i \in \mathcal{U}$. From (18) we know that this holds for all $i \in \mathcal{U}^{\mathrm{p}}$ and thus we only need to show this for all $i \in \mathcal{U}^{\mathrm{s}}$. Since $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} > 1$, we conclude that $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^* > p_i^*$. From this and from (19), we see $p_i^* = \min\left\{\overline{p}_i, \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^*, \widehat{\gamma}_i R_i(\mathbf{p}^*)\right\} = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\}$ for all $i \in \mathcal{U}^{\mathrm{s}}$, which completes the proof. ■

In the following Lemma, we derive the key properties of the fixed-points of our proposed power-update function.

Lemma 3: Our proposed algorithm guarantees zero-outage ratio for PUs, i.e., given any fixed-point $\mathbf{p}^*$ of the power-update function of the TPC-PP, we have $O^{\mathrm{p}}(\mathbf{p}^*) = 0$.

Proof: From **Theorem 2** we have $\varphi_k(\mathbf{p}^*) \leq \overline{\varphi}_k$ for all $k \in \mathcal{B}^{\mathrm{p}}$, and thus $\widehat{\gamma}_i R_i(\mathbf{p}^*) = \frac{\widehat{\theta}_i}{h_{b_i,i}} \varphi_{b_i}(\mathbf{p}^*) \leq \frac{\widehat{\theta}_i}{h_{b_i,i}} \overline{\varphi}_{b_i}$ holds for all $i \in \mathcal{U}^{\mathrm{p}}$. Furthermore, since $\overline{\varphi}_{b_i} = \min_{j\in\mathcal{U}^{\mathrm{p}}_{b_i}}\left\{\frac{\overline{p}_j h_{b_i,j}}{\widehat{\theta}_j}\right\} \leq \frac{\overline{p}_i h_{b_i,i}}{\widehat{\theta}_i}$, for all $i \in \mathcal{U}^{\mathrm{p}}$, we have $\frac{\widehat{\theta}_i}{h_{b_i,i}} \overline{\varphi}_{b_i} \leq \overline{p}_i$, from which we conclude $\frac{\widehat{\theta}_i}{h_{b_i,i}} \overline{\varphi}_{b_i} \leq \overline{p}_i$. Thus, for all $i \in \mathcal{U}^{\mathrm{p}}$, we have $\widehat{\gamma}_i R_i(\mathbf{p}^*) \leq \overline{p}_i$ from which we have $p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\} = \widehat{\gamma}_i R_i(\mathbf{p}^*)$. This proves that $\gamma_i(\mathbf{p}^*) = \widehat{\gamma}_i$ for all $i \in \mathcal{U}^{\mathrm{p}}$, or equivalently $O^{\mathrm{p}}(\mathbf{p}^*) = 0$. ■

The key properties of our proposed uplink power control algorithm are summarized as follows.

(a) Our proposed algorithm keeps the total received power plus noise at each PBS bellow the threshold given by (10) so that all the PUs attain their target-SINRs. In other words, the TPC-PP guarantees that the existence of the SUs does not cause outage to any PU. When the system is infeasible, all the PUs together with some SUs attain their target-SINRs, and the remaining SUs are unable to obtain their target-SINRs.

(b) When the system is feasible, the fixed-point of TPC-PP is unique and the same as that of the TPC power update function, at which all users attain their target-SINRs consuming minimum aggregate transmit power.

VI. Improved TPC-PP (ITPC-PP)

Although all fixed-points of the power-update function in the proposed TPC-PP algorithm result in zero-outage ratios for PUs, the outage ratios for SUs are not necessarily the same for all fixed-points. Among all possible fixed-points of the TPC-PP algorithm, the fixed-points with minimal outage ratio of SUs would be most desirable. The TPC-PP algorithm may converge to any of its fixed-points, depending of its initial transmit power vector. Now, an important question is how to lead the TPC-PP to converge to a desired fixed-point.

According to TPC-PP power update function in (12), when $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))} < 1$ at any iteration t, where $l = \arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$, each SU, whether it has high or low path-gain with PBS l, decreases its transmit power in proportion to $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))}$ to make the interference caused by SUs to PBSs lower than the threshold value. However, it is more efficient if the SUs, which cause more interference to PBS l (such SUs have high channel gains to PBS l), decrease their transmit power levels more than the other SUs. Thus, if an SU causes a very low interference to PBS l (such an SU has low channel gain with PBS l), that SU should not decrease its transmit power. This is because, reduction in its power may make it unsupported while not reducing the interference caused to the PBS l significantly. Accordingly, we propose the following improved TPC-PP (ITPC-PP) power update-function:

$$p_i(t+1) = \begin{cases} \min\left\{\overline{p}_i, \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}, & \text{for all } i \in \mathcal{U}^{\mathrm{p}} \\ \min\left\{\overline{p}_i, \beta_i(t) p_i(t), \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}, & \text{for all } i\in\mathcal{U}^{\mathrm{s}} \end{cases} \tag{20}$$

where

$$\beta_i(t)=\begin{cases}\beta(t), & \text{if } \beta(t)\geq 1\\ \beta(t)\left(1+|\overline{\varphi}_l-\varphi_l(\mathbf{p}(t))|\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}}\right), & \text{if } \beta(t)<1\end{cases} \tag{21}$$

In (21), $\beta(t)=\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$ and $l=\arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\{\overline{\varphi}_k/\varphi_k(\mathbf{p}(t))\}$.

From the viewpoint of signalling overhead, in ITPC-PP, in addition to the information required in TPC-PP, each SU needs to know (estimate) its channel gain with PBS l. In fact, the only difference between ITPC-PP and TPC-PP is that when $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\hat{\gamma}))}\right\}<1$, ITPC-PP causes each SU i to decrease its transmit power level in proportion to $\beta(t)\left(1+|\overline{\varphi}_l-\varphi_l(\mathbf{p}(t))|\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}}\right)$. On the other hand, in TPC-PP, all SUs decrease their transmit powers in proportion to $\beta(t)$. If the effective interference experienced by a given SU i at PBS l is lower than that of SU j, i.e., if $\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}}<\frac{\varphi_l(\mathbf{p}(t))-p_j(t)h_{l,j}}{h_{l,j}}$, the channel gain of SU i toward PBS l is better than that of SU j, and consequently, SU i causes more interference toward PBS l as compared to SU j. In this case, if $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))}<1$, SU i should reduce its transmit power more than SU j. This is done by adjusting $\beta_i(t)$ according to (21), because $\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}}<\frac{\varphi_l(\mathbf{p}(t))-p_j(t)h_{l,j}}{h_{l,j}}$ results in $\beta_i(t)<\beta_j(t)$ which causes SU i to decrease its power more in comparison with SU j. Therefore, with the proposed ITPC-PP, the SUs close to PBS l reduce their transmit power more as compared to SUs far from PBS l.

Theorem 3: Any fixed-point $\mathbf{p}^*$ for the TPC-PP power-update function (12) is also a fixed-point for the ITPC-PP power-update function (20).

Proof: Given a fixed-point $\mathbf{p}^*$ of the TPC-PP power-update function, from **Theorem 2**, we know that $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\}\geq 1$. Thus, $\mathbf{p}^*$ is also a fixed-point for the ITPC-PP power-update function in (20), because when $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\hat{\gamma}))}\right\}\geq 1$, we have $f_i^{\mathrm{ITPC-PP}}(\mathbf{p}^*)=f_i^{\mathrm{TPC-PP}}(\mathbf{p}^*)$ where $f_i^{\mathrm{TPC-PP}}(\mathbf{p})$ and $f_i^{\mathrm{ITPC-PP}}(\mathbf{p})$ are the power-update functions of TPC-PP and ITPC-PP, respectively. ■

Note that although any fixed-point of TPC-PP is also a fixed-point of ITPC-PP, when $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))}<1$, since ITPC-PP causes the SUs with high channel gains toward PBS l decrease their transmit power levels more aggressively, a fixed-point with improved outage ratio for SUs is eventually reached for ITPC-PP, while zero-outage ratio for PUs is still guaranteed. This will be demonstrated via the simulation results presented in Section VIII-B.

VII. Distributed Power Allocation Under Channel Uncertainties

The distributed power control approaches discussed in preceding sections are based upon the assumption that perfect channel information is known to the receivers, which may not be the case in practice. Therefore, in the following, we modify the power update equations for the TCP-PP algorithm considering uncertainties in channel gains. For this, we approximate the channel gain variations using ellipsoidal uncertainty sets [15]. We refer to the modified algorithm as robust TCP-PP (RTCP-PP).

A. Uncertainty Sets

Let us define the normalized channel gain of user i assigned to BS b_i as follows:

$$F_{i,j}=\begin{cases}\frac{h_{b_i,j}}{h_{b_i,i}}, & \text{if } i\neq j\\ 0, & \text{otherwise.}\end{cases} \tag{22}$$

We model the imperfect channel gains as

$$\tilde{F}_{i,j}=F_{i,j}+\Delta F_{i,j},\ \forall i,j\in\mathcal{U} \tag{23}$$

$$\tilde{h}_{k,i}=h_{k,i}+\Delta h_{k,i},\ \forall k\in\mathcal{B}, i\in\mathcal{U}, \tag{24}$$

where $\tilde{F}_{i,j}$ and $\tilde{h}_{k,i}$ are the actual (or uncertain) value obtained from nominal (or estimated) gains and corresponding perturbation part, e.g., $\Delta F_{i,j}$ and $\Delta h_{k,i}$, respectively. Without loss of generality, let $\mathbf{F}_i=[F_{i,j}]_{\forall j\in\mathcal{U}}$ and $\mathbf{H}_k=[h_{k,i}]_{\forall i\in\mathcal{U}}$ denote the normalized channel gain vector for user $i\in\mathcal{U}$ and the channel gain vector for BS $k\in\mathcal{B}$, respectively. Likewise, $\Delta\mathbf{F}_i$ and $\Delta\mathbf{H}_k$ represent the corresponding perturbation vectors. We approximate the uncertainties in the vector $\mathbf{F}_i$ and $\mathbf{H}_k$ due to fluctuations of the wireless link gains by ellipsoids. Let ξ_{F_i} and ξ_{H_k} represent the maximal deviation of each entries in $\mathbf{F}_i$ and $\mathbf{H}_k$. Under ellipsoidal approximation, the corresponding uncertainty sets $\widetilde{\mathcal{F}}_i$ and $\widetilde{\mathcal{H}}_k$ for $\mathbf{F}_i$ and $\mathbf{H}_k$, respectively, can be written as

$$\widetilde{\mathcal{F}}_i=\left\{\mathbf{F}_i+\Delta\mathbf{F}_i:\sum_{j\neq i}|\Delta F_{i,j}|^2\leq\xi_{F_i}^2\right\},\ \forall i\in\mathcal{U} \tag{25}$$

$$\widetilde{\mathcal{H}}_k=\left\{\mathbf{H}_k+\Delta\mathbf{H}_k:\sum_{i\in\mathcal{U}}|\Delta h_{k,i}|^2\leq\xi_{H_k}^2\right\},\ \forall k\in\mathcal{B}. \tag{26}$$

Using the uncertainty set $\widetilde{\mathcal{F}}_i$ the SINR expression in (1) can be equivalently written as follows [16], [17]:

$$\breve{\gamma}_i(\mathbf{p})\triangleq\frac{p_i}{\sum_{j\in\mathcal{U},j\neq i}p_j(F_{i,j}+\Delta F_{i,j})+\tilde{\sigma}_{b_i}^2}, \tag{27}$$

where $\tilde{\sigma}_{b_i}^2=\frac{\sigma_{b_i^2}}{h_{b_i,i}}$. Likewise, the total interference power at BS $k\in\mathcal{B}$ given by (2) can be written as

$$\breve{\varphi}_k(\mathbf{p})=\sum_{i\in\mathcal{U}}p_i(h_{k,i}+\Delta h_{k,i})+\sigma_k^2. \tag{28}$$

Utilizing the Cauchy-Schwarz inequality [18], we obtain,

$$\sum_{j\in\mathcal{U},j\neq i}p_j\Delta F_{i,j}\leq\sqrt{\sum_{j\in\mathcal{U},j\neq i}|p_j|^2\sum_{j\in\mathcal{U},j\neq i}|\Delta F_{i,j}|^2}$$
$$\leq\xi_{F_i}\sqrt{\sum_{j\in\mathcal{U},j\neq i}p_j^2}. \tag{29}$$

Similarly,

$$\sum_{i\in\mathcal{U}} p_i \Delta H_{k,i} \leq \xi_{H_k} \sqrt{\sum_{i\in\mathcal{U}} p_i^2}. \tag{30}$$

From (29) and (30), we can rewrite (27) and (28) under channel uncertainties as follows:

$$\widetilde{\gamma}_i(\mathbf{p}) = \frac{p_i}{\sum_{j\in\mathcal{U},j\neq i} p_j F_{i,j} + \xi_{F_i}\sqrt{\sum_{j\in\mathcal{U},j\neq i} p_j^2} + \tilde{\sigma}_{b_i}^2} \tag{31}$$

$$\widetilde{\varphi}_k(\mathbf{p}) = \sum_{i\in\mathcal{U}} p_i h_{k,i} + \xi_{H_k}\sqrt{\sum_{i\in\mathcal{U}} p_i^2} + \sigma_k^2. \tag{32}$$

B. Iterative Power Update Under Channel Uncertainty

For any time instance t, let us define the parameter $\mathfrak{Q}(t) = \sqrt{\sum_{i\in\mathcal{U}} p_i^2(t)}$. Then the power update functions are given by (33), (See equation at the bottom of the page) where $\widetilde{\beta}(t) = \min_{k\in\mathcal{B}^{\mathrm{p}}} \left\{\frac{\overline{\varphi}_k}{\widetilde{\varphi}_k(\mathbf{p}(t))}\right\}$.

Note that the power update functions in (33) for RTCP-PP are similar to those for the TCP-PP algorithm with the modified SINR expression $\widetilde{\gamma}_i(\mathbf{p})$ and received-power-temperature ratio $\widetilde{\beta}(t)$ as well as an additive term. This additive term $\xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)}$ for $\forall i \in \mathcal{U}$ is referred to as *protection function* [15], [19] against uncertainties. The users broadcast their transmit powers every time slot, from which the BSs independently calculate the parameter $\mathfrak{Q}(t)$ and multicast this to the corresponding users. Hence the users can update the power independently similar to **Algorithm 1**. If the uncertainty parameters ξ_{F_i}, ξ_{H_k} for $\forall i,k$ become zero, RTCP-PP reduces to the TPC-PP algorithm, i.e., no channel uncertainty is taken into consideration.

The RTCP-PP algorithm is robust against channel uncertainties since it considers the uncertainties ahead of time, which are deterministically calculated from the realizations of the uncertain parameters to certain extent (i.e., a bounded error region). The algorithm therefore becomes robust to the channel uncertainties at the cost of some performance degradation (which will be explained in the Section VIII-C). As the bounds (i.e., ξ_{F_i}, ξ_{H_k}) become higher, the system becomes more robust against channel uncertainties. However, larger values of the bounds may affect the performance (e.g., achievable SINR, outage ratio etc.) significantly.

VIII. Simulation Results

We present numerical results to illustrate the performances of our proposed TPC-PP, ITPC-PP, and RTPC-PP algorithms and compare them with that of the TPC algorithm. The uplink channel gain from each user i to each BS k is given by $0.1d_{k,i}^{-3}$ where $d_{k,i}$ is the distance between user i and BS k. The upper bound on the transmit power for all users is 1 Watt. We first consider a single snapshot of locations of users and BSs in the network to obtain insight into how TPC-PP works in comparison with the TPC, and then proceed to different snapshots of users' and BSs' locations to verify that the results do not depend on specific user-locations. In Sections VIII-A and VIII-B we show the numerical results assuming that perfect CSI is available to the receivers. Section VIII-C demonstrates the performance results under channel uncertainty.

A. Single Snapshot Scenario

Let us consider a network where 6 PUs and 6 SUs are fixed and served by two PBSs and two SBSs, respectively, in an area of 1000 m $\times$ 1000 m, as illustrated in Fig. 1. In this network, each primary (secondary) BS serves 3 primary (secondary) users. For simplicity, suppose that the target SINRs for all PUs and SUs is 0.20. The simulation results for two cases in which users iteratively update their transmit power levels using TPC or TPC-PP, respectively, are shown in Figs. 2–4. Fig. 2 illustrates the total received power plus noise for each PBS normalized by its corresponding total received-power-temperature, i.e., $\frac{\varphi_k(\mathbf{p})}{\overline{\varphi}_k}$, $k \in \mathcal{B}^{\mathrm{p}}$, versus iteration number, for TPC and TPC-PP. The transmit power levels and SINRs for SUs and PUs are shown in Fig. 3(a) and (b) and Figs. 3(a)–4(b) for TPC and TPC-PP, respectively. When the TPC is employed, the total received power at PBS 1 exceeds its maximum received power-temperature, and thus zero-outage ratio for PUs connecting to this PBS is not guaranteed, as shown in Figs. 2 and 3(b). However, when TPC-PP is employed, the total received power at each PBS does not exceed its corresponding maximum received-power-temperature (see Fig. 2), which guarantees zero-outage ratio for PUs (see Fig. 4(b)). Furthermore, by employing TPC-PP, at the equilibrium, we have $\varphi_1(\mathbf{p}) = \overline{\varphi}_1$ for PBS 1, as it was shown in **Theorem 2**. More specifically, as seen in Figs. 4(a) and 3(b), by employing TPC, 4 users including two PUs and two SUs are unable to reach their target-SINRs, whereas by employing TPC-PP, only 3 users are unsupported and these users do not include any PU (see Fig. 4(a) and (b)). This demonstrates that TPC-PP not only guarantees zero-outage ratio for PUs (as shown in **Lemma 3**), but also improves the number of supported SUs.

$$p_i(t+1) = \begin{cases} \min\left\{\overline{p}_i, \frac{\widehat{\gamma}_i}{\widetilde{\gamma}_i(\mathbf{p}(t))}\left(p_i(t) + \xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)}\right)\right\}, & \forall i \in \mathcal{U}^{\mathrm{p}} \\ \min\left\{\overline{p}_i, \widetilde{\beta}(t)\left(p_i(t) + \xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)}\right), \frac{\widehat{\gamma}_i}{\widetilde{\gamma}_i(\mathbf{p}(t))}\left(p_i(t) + \xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)}\right)\right\}, & \forall i \in \mathcal{U}^{\mathrm{s}}. \end{cases} \tag{33}$$

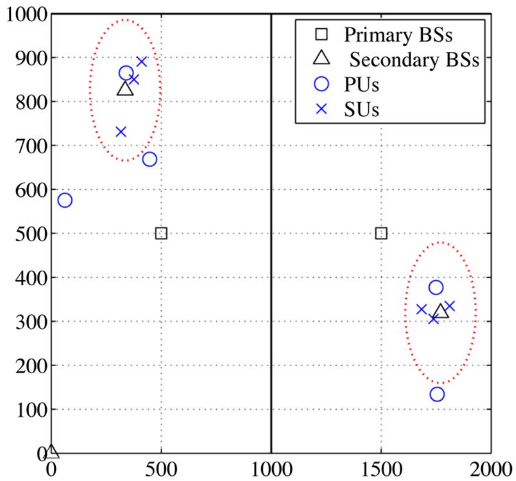

Fig. 1. Network topology and the placement of users and base stations.

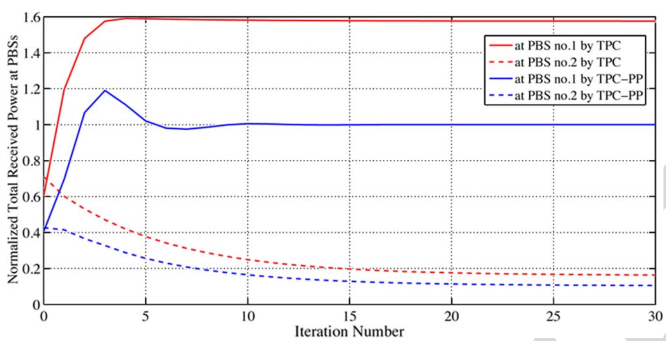

Fig. 2. Normalized total received power plus noise for each PBS versus iteration number, defined as $\frac{\varphi_k(\mathbf{p}(t))}{\overline{\varphi}_k}$ for PBS k, where $k = \{1,2\}$, for the TPC and TPC-PP algorithms.

B. Different Snapshots

Now, we compare the performances of TPC-PP, ITPC-PP, and TPC for different snapshots of users' locations and for different values of target-SINRs. For benchmarking purpose, we also compare the performance of our proposed distributed algorithms with a centralized approach called the link gain ratio-based algorithm (LGR) proposed in [2].

Unlike the existing centralized joint power and admission control algorithms, LGR predetermines the admission order of secondary users based on a the link gain ratio metric defined as $\min_{k\in\mathcal{B}^{\mathrm{P}}}\{\overline{I}_k \frac{h_{b_i,i}}{h_{k,i}}\}$, where $\overline{I}_k$ is the interference temperature. In [2], $\overline{I}_k$ is assumed to be fixed, whereas in our case it is dynamic as discussed in Section IV-A and it is given by (11). For this reason, for simulating LGR, we use the total received-power-temperature instead of the interference temperature, i.e., the LGR metric $\min_{k\in\mathcal{B}^{\mathrm{P}}}\{\overline{\varphi}_k \frac{h_{b_i,i}}{h_{k,i}}\}$ is adopted. Using bisection search, the LGR algorithm admits as many secondary users as possible with highest LGRs (or equivalently, removes as few secondary users as possible with lowest LGRs) so that the target-SINRs for all the PUs and the remaining SUs get feasible. The complexity of LGR algorithm is of $O(|\mathcal{U}_s|^2 \log |\mathcal{U}_s|)$ [2], where $|\mathcal{U}_s|$ is the total number of SUs. A drawback of LGR algorithm is that it does not consider different values of the target-SINRs in the admission of the secondary users. Different

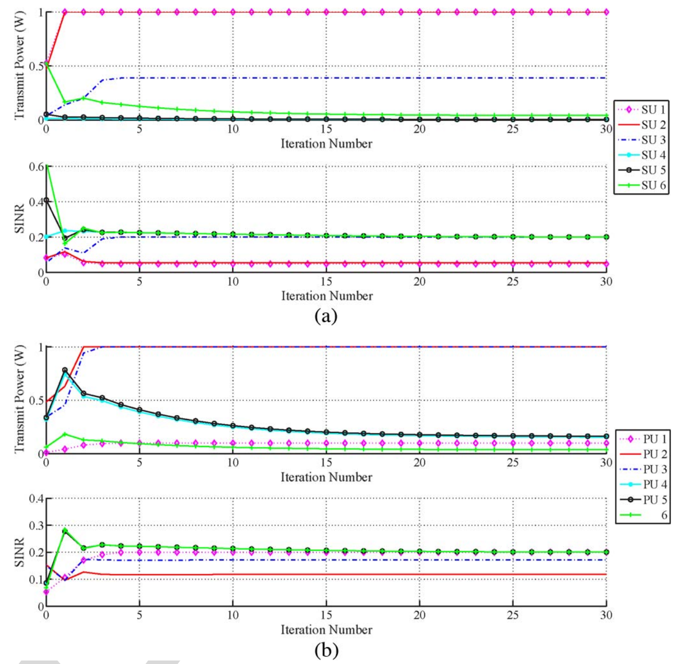

Fig. 3. Transmit power and SINR versus iteration for the TPC algorithm: (a) for SUs, (b) for PUs.

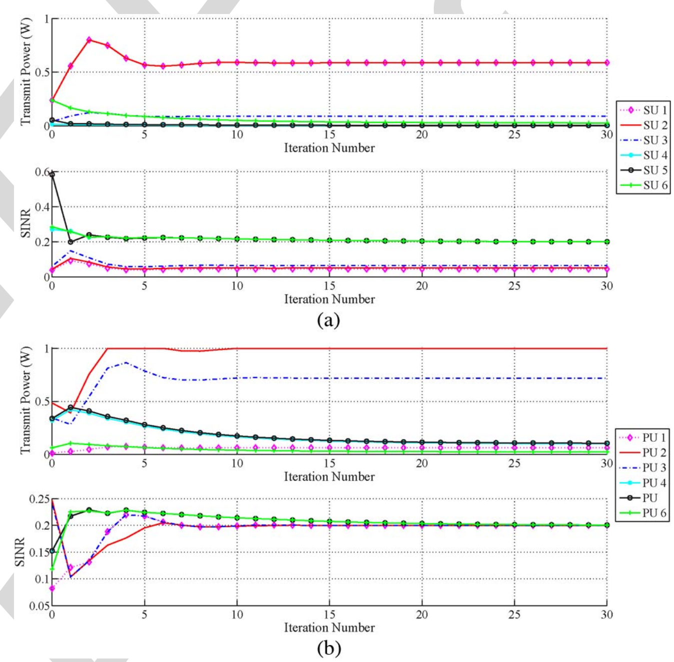

Fig. 4. Transmit power and SINR versus iteration for the TPC-PP algorithm: (a) for SUs, (b) for PUs.

target-SINRs are possible in networks where different applications are used by different users.

Let us consider a primary network with 3×3 cells where each primary cell covers an area of 1000 m $\times$ 1000 m. Each primary (secondary) user is associated with only one primary (secondary) BS. Each PBS is located at the centre of its corresponding cell and serves 5 PUs. Within this primary network of 3×3 cells, we consider a secondary radio network under two scenarios, namely, with *small cells* (i.e., cells with small transmission radius) and with *large cells* (i.e., cells with larger

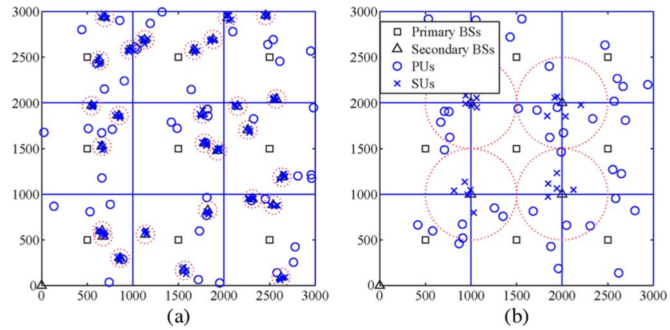

Fig. 5. An example of network topology for a primary network with 3×3 cells with 5 PUs per primary cell, which coexists with a secondary network with small cells [Fig. 5(a)] and large cells [Fig. 5(b)]. The network in Fig. 5(a) includes 3 secondary BSs within each primary cell and 5 SUs per each secondary BS, and the network in Fig. 5(b) includes 4 secondary BSs and 5 SUs per each secondary BS.

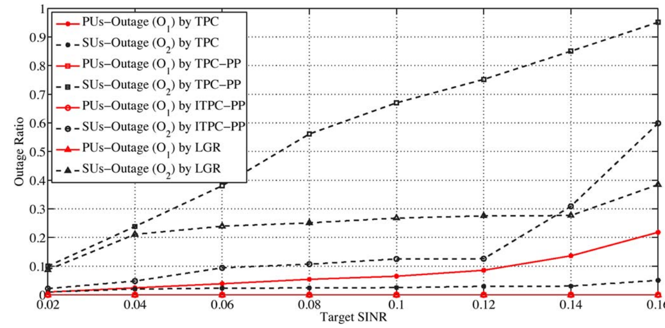

Fig. 6. Average Outage ratios for PUs (O_1) and for SUs (O_2) versus different values of target-SINRs for TPC, TPC-PP, ITPC-PP, and LGR in a small cell CRN. Note that $O_1 = 0$ for TPC-PP, ITPC-PP, and LGR.

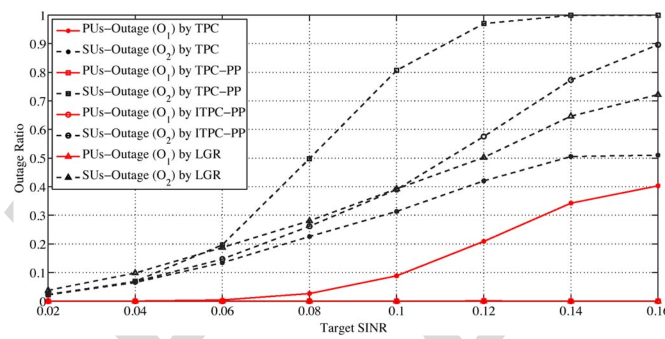

Fig. 7. Average Outage ratios for PUs (O_1) and for SUs (O_2) versus different values of target-SINRs for TPC, TPC-PP, ITPC-PP, and LGR in a large cell CRN. Note that $O_1 = 0$ for TPC-PP, ITPC-PP, and LGR.

transmission radius). The target-SINRs are considered to be the same for all users, ranging from 0.02 to 0.16 with step size of 0.02. Since for values of target-SINR higher than 0.16, the target-SINRs for PUs become infeasible, we use 0.16 as the upper limit of the target-SINR of users. For each target-SINR, we average the corresponding values of outage ratios for the PUs and SUs for TPC, TPC-PP, ITPC-PP, and LGR algorithms for 1000 independent snapshots for a uniform distribution of BSs and users' locations. The initial transmit power for each user is uniformly set from the interval [0,1] for each snapshot.

1) Secondary Radio Network With Small Cells: At each primary cell, 3 secondary BSs are uniformly located, each of which serves 8 SUs uniformly located at a radius of 200 m around it. Thus, the entire network consists of 9 PBSs, 45 PUs, 27 secondary BSs, and 135 SUs. An example of such a network setting is shown in Fig. 5(a). Fig. 6 shows the average outage ratio versus target-SINR, for TPC, TPC-PP, ITPC-PP, and LGR, over 1000 independent snapshots of uniformly distributed locations of users and secondary BSs. Note that both TPC-PP and ITPC-PP outperform TPC with respect to the capability of guaranteeing a zero-outage ratio for PUs at the cost of increased outage ratio for SUs. Moreover, when the total interference caused by SUs to PBS is higher than the threshold, in ITPC-PP, SUs, which cause more interference to PBSs, reduce their transmit power levels. This results in a lower outage ratio for the SUs as compared to the TPC-PP in which all SUs reduce their transmit power levels. For instance, with target-SINR of 0.12, by using TPC-PP and ITPC-PP, the outage ratio for the SUs is 0.75, and 0.12, respectively. When the target-SINR is increased, for example, with target-SINR of 0.16, more SUs have to be removed to keep the total received power below the total received-power-temperature (which is low due to high-SINR requirement by PUs). In other words, the lower and higher values of the SINR requirements for the PUs result in higher and lower values of total received-power-temperature at PBSs, respectively, which correspond to the underlay and the overlay spectrum access strategies used by TPC-PP and ITPC-PP. Thus TPC-PP and ITPC-PP adaptively use a mixed-strategy for spectrum access. Furthermore, ITPC-PP and TPC-PP result in zero-outage ratio for PUs as in LGR, and ITPC-PP follows the outage ratio for SUs as obtained by the centralized LGR algorithm. Specifically, for lower values of target-SINRs, the ITPC-PP algorithm is superior to the LGR algorithm, but it is inferior for higher values of target-SINRs. This shows that our proposed distributed algorithm has comparable performance to that of the centralized LGR algorithm, however at a much lower complexity.

2) Secondary Radio Network With Large Cells: Now consider a CRN with 4 large-cells each of which serves 5 SUs uniformly located at a radius of 1000 m around it within the coverage area of the 3×3 cells primary network. Thus, the entire network consists of 9 PBSs, 45 PUs, 4 secondary BSs, and 20 SUs. An example of such a network setting is shown in Fig. 7(b). Fig. 7 shows the average outage ratio versus target-SINR, for TPC, TPC-PP, and ITPC-PP, over 1000 independent snapshots of uniformly distributed locations of users. Similar to the secondary network setting with small cells, ITPC-PP outperforms TPC-PP in terms of outage ratio for SUs, and both outperform TPC with respect to the capability of guaranteeing a zero-outage ratio for PUs at the cost of increased outage ratio for SUs. Also, the ITPC-PP algorithm follows the outage ratios of PUs and SUs obtained by the centralized LGR algorithm.

In Fig. 8, we illustrate the average rate of convergence of TPC, TPC-PP and ITPC-PP algorithms for both the small cell and large cell scenarios explained above. The rate of convergence $\tau(t)$ at iteration t is measured as normalized Euclidean distance of transmit power, e.g., $\tau(t) = \frac{\|\mathbf{p}(t)-\mathbf{p}^*\|_2}{\|\mathbf{p}(0)-\mathbf{p}^*\|_2}$, where $\mathbf{p}(0)$ is the initial transmit power vector, $\mathbf{p}(t)$ is the transmit power vector at iteration t, $\mathbf{p}^*$ is the fixed-point [corresponding to

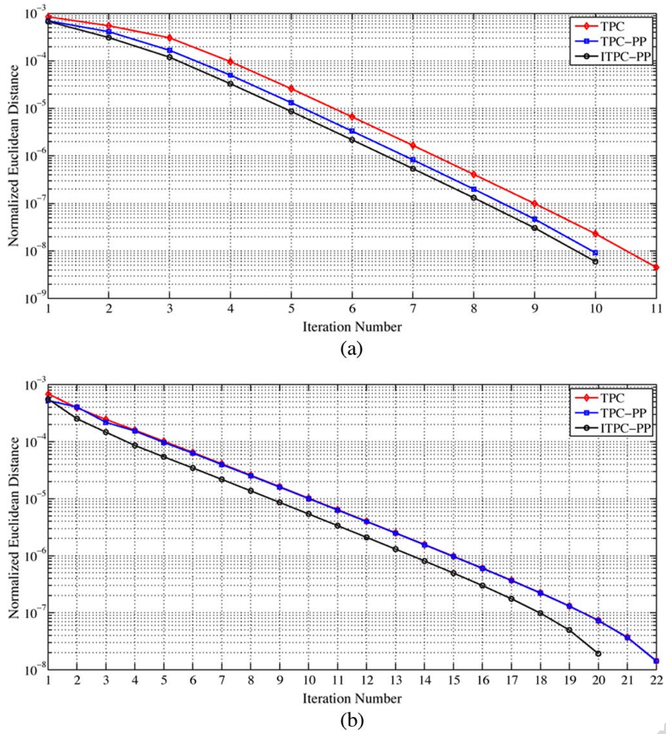

Fig. 8. Rate of convergence versus iteration for the TPC, TPC-PP and ITPC-PP algorithms: (a) secondary network with small cells and (b) secondary network with large cells. The rate of convergence is measures as normalized Euclidean distance of transmit power, which is given by $\frac{\|\mathbf{p}(t)-\mathbf{p}^*\|_2}{\|\mathbf{p}(0)-\mathbf{p}^*\|_2}$.

the initial transmit power vector $\mathbf{p}(0)$] to which the algorithm converges, and $\|.\|_2$ denote the Euclidean norm. We select the target-SINR for each PU and SU randomly from the set of {0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14} and average the results over 1000 independent simulation realizations. The rest of the simulation parameters are the same as those mentioned for small cell and large cell scenarios. As can be seen from Fig. 8, the rate of convergence of our proposed algorithms is improved in comparison with that of the TPC, which is known to be a fast convergent distributed power control algorithm. In particular, TPC-PP outperforms TPC for small cell scenarios and provides similar convergence rate for large cell scenarios. ITPC-PP outperforms TPC and TPC-PP both for small cell and large cell scenarios.

C. Performance Under Channel Uncertainty

In the following, we observe the performance of our proposed algorithms considering the uncertainty in channel gains. We measure the uncertainty in channel gains as percentages and assume the similar uncertainty bounds in the CSI values for all users. For example, uncertainty bound $\xi = \xi_{F_i} = \xi_{H_k} = 0.02$ means that estimation error in the CSI values $\mathbf{F}_i$ and $\mathbf{H}_k$, $\forall i,k$ is not more than 2% of their nominal values. The numerical results are averaged over 200 independent network realizations. The target-SINRs for all PUs and SUs are set to 0.10 and the rest of the simulation parameters are same as those mentioned in Section VIII-A. Different power control schemes used in the simulations to observe the performance under channel uncertainties are summarized in Table II.

TABLE II
POWER CONTROL SCHEMES USED FOR PERFORMANCE COMPARISON UNDER CHANNEL UNCERTAINTY

Scheme	Update Expression	Ref. Figure(s)
1. TPC (perfect CSI)	(6)	Fig. 11
2. TPC-PP (perfect CSI)	(12)	Fig. 11
3. TPC (uncertain CSI)	(6) with $\gamma_i(\mathbf{p}(t))$ is replaced by $\widetilde{\gamma}_i(\mathbf{p}(t))$	Figs. 9-11
4. TPC-PP (uncertain CSI)	(12) with $\gamma_i(\mathbf{p}(t))$ and $\beta(t)$ are replaced by $\widetilde{\gamma}_i(\mathbf{p}(t))$ and $\widetilde{\beta}(t)$, respectively	Figs. 9-11
5. RTPC	(34)	Figs. 9-11
6. RTPC-PP	(33)	Figs. 9-11

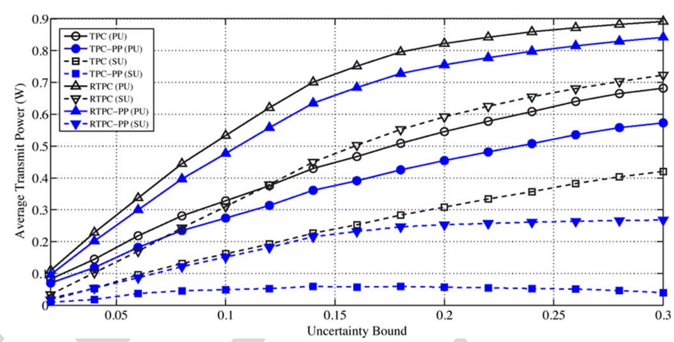

Fig. 9. Average transmit power versus uncertainty bound in TPC, TPC-PP, RTPC, and RTPC-PP algorithm under imperfect CSI.

Note that under channel uncertainties, the TPC power update expression (referred to as robust TPC [RTPC]) for all $i \in \mathcal{U}$ is given by

$$p_i(t+1)=\min\left\{\overline{p}_i, \frac{\widehat{\gamma}_i}{\widetilde{\gamma}_i(\mathbf{p}(t))}\left(p_i(t)+\xi_{F_i}\sqrt{\mathfrak{Q}^2(t)-p_i^2(t)}\right)\right\}. \tag{34}$$

In Fig. 9, considering uncertainty in CSI, we plot the average transmit power[2] for the PUs and SUs with uncertainty bound for both the RTPC and RTPC-PP algorithms. Note that with uncertain CSI values, as mentioned in third and fourth rows of Table II, the parameters $\gamma_i(\mathbf{p}(t))$ and $\beta(t)$ in the power update expression of TPC and TPC-PP algorithm will be replaced with $\widetilde{\gamma}_i(\mathbf{p}(t))$ and $\widetilde{\beta}(t)$, respectively. When the uncertainty bound increases, the PUs increase the transmit power to achieve target-SINR.

Under channel uncertainties, higher uncertainty bounds imply higher fluctuations in CSI values and hence the users require higher transmit powers to overcome the impact of channel uncertainty. Although both the PUs and SUs increase the power in RTPC (TPC) algorithm, RTPC-PP (TPC-PP) prevents the SUs from increasing the power using the parameter $\widetilde{\beta}$ and hence transmit power of SUs are less in RTPC-PP (TPC-PP) compared to RTPC (TPC) which also minimizes the effect of interference from SUs. Another interesting observation is that as the uncertainty bound keeps increasing, the total power approaches to the upper limit $\overline{p}_i$.

[2]The average transmit powers for the PUs and SUs are given by $\frac{\sum p_i}{|\mathcal{U}^P|}$ and $\frac{\sum p_i}{|\mathcal{U}^S|}$, respectively. Similarly, the average SINRs for PUs and SUs are calculated as $\frac{\sum \gamma_i(\mathbf{p})}{|\mathcal{U}^P|}$ and $\frac{\sum \gamma_i(\mathbf{p})}{|\mathcal{U}^S|)}$.

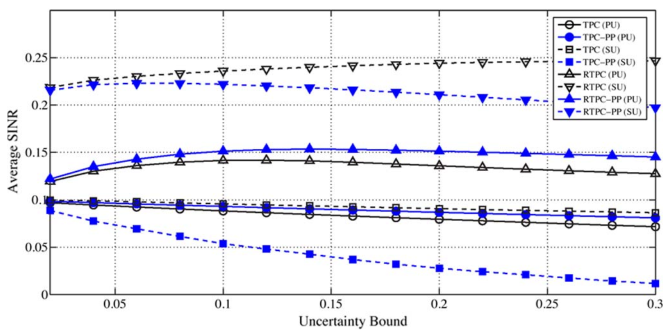

Fig. 10. Average SINR versus uncertainty bound in TPC, TPC-PP, RTPC, and RTPC-PP algorithm under imperfect CSI.

The impact of higher transmit power on users' achievable SINR under channel uncertainties is shown in Fig. 10. As we have seen in Fig. 9, users need to increase their transmit powers to achieve target-SINR. However, higher transmit powers cause more interference at the BSs and hence the SINR decreases at higher uncertainty bounds. Besides, since TPC-PP[3] and TPC do not consider any channel uncertainties, as the uncertainty bound increases, SINR of TPC-PP (TPC) decreases significantly compared to RTPC-PP (RTPC). This is due to the fact that RTPC-PP (RTPC) provides robustness against uncertainties by means of protection function and the users update their power accordingly to achieve target-SINR. Recall that, the RTPC algorithm does not consider interference temperature at the PBSs. Hence, the SUs increase their transmit powers to overcome channel uncertainties, which causes severe interference to PBSs and the SINR for the PUs decreases significantly compared to the proposed RTPC-PP algorithm. In addition, as the transmit power of all the users reaches to its maximum limit, increasing the uncertainty bounds reduces SINR. Note that the SINR expression in (31) using protection function can be written as $\widetilde{\gamma}_i(\mathbf{p}) = \frac{p_i}{\sum_{j\in\mathfrak{U}, j\neq i} p_j F_{i,j} + \xi\sqrt{\Omega^2 - p_i^2} + \tilde{\sigma}_{b_i}^2}$, where $\xi = \xi_{F_i}, \forall i$. When the users transmit with their maximum power, the term $\sqrt{\Omega^2 - p_i^2}$ becomes fixed, and consequently, higher bounds (e.g., higher ξ values) decrease the SINR.

The outage ratios for RTPC, RTPC-PP, TPC, and TPC-PP algorithms are shown in Fig. 11. With perfect CSI at the receivers, the expressions for power update for TPC and TPC-PP are given by (6) and (12), respectively. Note that under imperfect CSI, since the users (both PUs and SUs) need to increase their transmit powers to overcome the impact of uncertainty, which causes more interference, the zero outage for PUs in RTPC-PP is not guaranteed. Under uncertain CSI, RTPC-PP (RTPC) outperforms TPC-PP (TPC) since TPC-PP (TPC) does not consider any channel uncertainties in power updates. Note that, RTPC-PP (TPC-PP) always outperforms TPC-PP (TPC) in terms of PU outage. Since TPC does not provide any protection for PUs, under uncertain CSI, the SUs increase their transmit powers to achieve their target-SINR. This leads to zero outage for SUs but significantly increases the outage of PUs. In addition, under perfect CSI, TPC and TPC-PP do not

[3]Recall that under uncertain CSI, the power update expression for TPC-PP (TPC) is given by fourth (third) row of Table II.

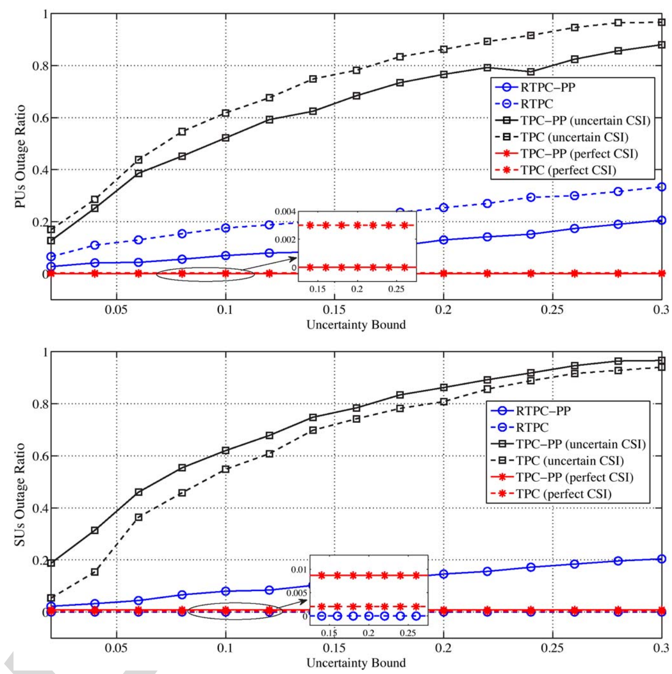

Fig. 11. Outage ratio versus uncertainty bound in RTPC, RTPC-PP, TPC, TPC-PP algorithm under perfect and imperfect CSI.

consider the channel variations, and the outage is independent of uncertainty bounds. With the perfect CSI values, the outage for PUs is always zero for TPC-PP at the cost of a higher outage for SUs, when compared to TPC.

Higher uncertainty bounds make the system more robust against channel fluctuations. However, as we have seen from Figs. 9–11, there is a trade-off between robustness and system performance since higher uncertainty bounds degrade the SINR and may increase the outage significantly.

IX. Conclusion

We have proposed distributed uplink power control algorithms (TPC-PP and ITPC-PP) for CRNs in multi-cell environments where the outage ratio for the SUs is minimized subject to the constraint of zero-outage ratio for the PUs. We have showed that our proposed distributed power-update functions corresponding to TPC-PP and ITPC-PP have at least one fixed-point. We have also showed that our proposed algorithms not only guarantee the zero-outage ratio for the PUs, but also enable the SUs to use a mixed-strategy adaptively for spectrum access to improve their outage ratio. Also, the performance of the proposed distributed ITPC-PP algorithm has been shown to be comparable to that of centralized LGR algorithm. However, the complexity of ITPC-PP is much lower than that of LGR. We have also provided a power control scheme (RTPC-PP) which provides robustness against channel uncertainties at the cost of a higher outage ratio compared to TPC-PP.

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Distributed Uplink Power Control for Multi-Cell Cognitive Radio Networks

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***Abstract*—We present a distributed power control algorithm to address the uplink interference management problem in cognitive radio networks where the underlaying secondary users (SUs) share the same licensed spectrum with the primary users (PUs) in multi-cell environments. Since the PUs have a higher priority of channel access compared to the SUs, minimal number of SUs should be gradually removed, subject to the constraint that all primary users are supported with their target signal-to-interference-plus-noise ratios (SINRs), which is assumed feasible. In our proposed algorithm, each primary user rigidly tracks its target-SINR by employing the conventional target-SINR tracking power control algorithm (TPC). Each transmitting SU employs the TPC as long as the total received power at the primary receiver is below a given threshold; otherwise, it decreases its transmit power in proportion to the ratio between the given threshold and the total received power at the primary receiver, which is referred to as the *total received-power-temperature*. We show that our proposed distributed power-update function has at least one fixed-point. We also show that our proposed algorithm not only improves the number of supported SUs but also guarantees that all primary users are supported with their (feasible) target-SINRs. Finally, we also propose an enhanced power control algorithm that achieves zero-outage for PUs and a better outage ratio for SUs. To this end, we provide a robust power control method that considers the uncertainties in channel gains.**

***Index Terms*—Cellular cognitive wireless networks, interference temperature limit, underlay and overlay spectrum access, mixed strategy spectrum access, distributed interference control, total received-power-temperature, uncertain channel state information, uncertainty sets.**

I. Introduction

In a cognitive radio network (CRN), secondary users (SUs) coexist with primary users (PUs) using spectrum overlay or spectrum underlay to exploit the radio spectrum licensed to PUs. In the overlay strategy, when a PU is active, no SU transmits (i.e., the interference temperature limit is assumed to be zero), and thus the interference tolerability of the primary network is ignored. On the other hand, in the pure underlay strategy, which uses a fixed interference temperature limit, the transmission opportunities of SUs during the idle periods (i.e., when no PU is active) are wasted [1]. Therefore, instead of assuming that the interference temperature limit is fixed, we propose that it can be varied dynamically in an optimum manner and a *mixed-strategy* can be adopted. In particular, the value of interference temperature at each primary receiver can be dynamically decreased (increased) as the number of its corresponding PUs is increased (decreased) and/or their channel status becomes weaker (stronger). For example, when many PUs with large target-signal-to-interference-plus noise ratio (SINR) requirements are active and/or the corresponding channel gains (from transmitters to the receivers) are poor, the interference temperature is set to a very small value (or even zero). This corresponds to the spectrum overlay strategy. On the other hand, when a number of PUs with moderate target-SINR requirements and/or good channel gains are active, a non-zero value of the interference temperature limit can be chosen such that the requirements of the PUs can still be satisfied. This corresponds to the spectrum underlay strategy.

By dynamically setting the value of the interference temperature limit, a mixed-strategy is obtained. With this mixed strategy, the spectrum access opportunities as well as the interference tolerability of the primary network, which are missed in the pure underlay and overlay strategies, respectively, can be exploited to improve the performance of SUs. This mixed strategy can be implemented through an efficent power control method. However, this power control problem is a non-deterministic polynomial-time (NP)-complete problem [2] and *centralized* algorithms have been proposed in [2], [3] to solve the problem sub-optimally. However, the signalling complexity of such algorithms could be high and these schemes might be useful for benchmarking purpose only.

In this paper, we address the problem of *distributed* uplink power control in cellular CRNs. Having obtained the interference temperature limit of each primary receiver, we aim to devise a distributed power control scheme for the PUs and SUs to set their transmit power levels so that a maximal number of SUs reach their target-SINRs, while all the PUs are supported with their target-SINRs (i.e., the interference caused by the SUs to each primary receiver remains below its interference temperature limit).

The existing distributed interference management algorithms in conventional cellular wireless networks do not guarantee that the total interference caused to PUs by SUs does not exceed a given threshold, which result in outage of some PUs (i.e., some

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PUs are not supported with their required SINRs). However, these algorithms can be used by the SUs, provided that the interference caused by them to the PUs does not exceed a given threshold. In particular, if the SUs limit their transmit power levels so that the total interference caused to the PUs does not exceed a given threshold (which each primary receiver can broadcast to all SUs), each PU is able to reach its target-SINR, and the SUs can minimize their outage ratios by employing an existing distributed power control algorithm. This is the idea that we use in this paper to develop distributed uplink power control algorithms. The contributions of this paper can be summarized as follows.

- We formally define the problem of uplink power control in CRNs in multicellular environments to minimize the outage ratio for the SUs subject to the zero-outage constraint for the PUs. We present a distributed power control scheme to achieve this design goal. Specifically, in our proposed algorithm, each PU rigidly tracks its target-SINR by employing the traditional TPC algorithm proposed in [7]. Each transmitting SU employs the TPC algorithm as long as the total received power at each of the primary receivers is below a given threshold; otherwise, it decreases its transmit power in proportion to the ratio of the given threshold to the total received power at a primary receiver. We refer to our proposed algorithm as TPC with PU-protection (TPC-PP).
- We prove that the proposed distributed power-update function corresponding to TPC-PP has at least one fixed-point. We also show that the proposed algorithm not only significantly decreases the outage ratio of SUs, but also guarantees zero-outage ratio for the PUs.
- We also devise an improved TPC-PP algorithm (called ITPC-PP), which achieves better outage ratios for SUs and zero-outage for PUs.
- Due to the stochastic nature of wireless channels we develop a power control algorithm that is resilient against channel fluctuations. We refer to this algorithm as robust TCP-PP (RTPC-PP). Through simulations we show that the RTPC-PP scheme is robust against channel uncertainties at the cost of a higher outage ratio compared to TPC-PP.
- Performances of the proposed algorithms are evaluated and also compared against a state-of-the-art centralized algorithm for uplink power control for cellular CRNs.

It is worth noting that emerging wireless networks such as the multi-tier cellular networks and/or device-to-device communication networks, face the same problem of prioritized uplink power control and interference management where all users in different tiers share the same licensed spectrum but with different priorities of access. Thus our proposed power control algorithms can also be employed in such networks for cross-tier interference management. Also, note that the proposed power control methods can be used for both orthogonal frequency-division multiple access (OFDMA) and code-division multiple access (CDMA)-based CRNs. While in the former case uplink power control is performed for transmission over different sub-channels shared among PUs and SUs over space and time, in the latter case, uplink power control is performed for transmission over the entire spectrum (i.e., a single channel).

The rest of this paper is organized as follows. Section II reviews the related literature and discusses the motivation and novelty of this work. In Section III, we introduce the system model and existing distributed power control algorithms, and present a formal statement of the interference management problem in CRNs. Section IV introduces our proposed distributed interference control algorithm. In Section V, we analyze the proposed method and derive its key properties. Section VI describes how the proposed algorithm can be improved. The power control algorithm under channel uncertainty is provided in Section VII. Simulation results are presented in Section VIII. Section IX concludes the paper.

II. Related Literature and Novelty of the Work

A few works in the literature have addressed the uplink power control and admission control problem in CRNs (Table I). These works are based on removal algorithms, whereby the least number of SUs are removed so that all admitted SUs obtain their target-SINRs and they do not cause outage to any PU. The removal algorithms generally consist of two phases, namely, the *feasibility checking phase* and the *removal phase*.

To check the feasibility of the constraints, the existing algorithms as in [4] use centralized techniques based on calculating spectral radius of a matrix of path-gains and target-SINRs developed in [5]. The algorithms in [3] and [6] use the TPC algorithm proposed in [7]. In [8], a random searching algorithm is proposed where probabilistic mechanisms are used for the SUs to access the channel. This algorithm may not converge and its performance depends on the initial starting point. In [9] and [3], sequential admission control algorithms are proposed in which, based on certain metrics, an opportunity for accessing the network is assigned to each of the SUs. Non-supported SUs with lower network access opportunity are sequentially removed until the remaining SUs along with all PUs reach a feasible power vector. In [2], assuming the same QoS (i.e., target-SINR) for all the SUs, an algorithm is proposed in which the SUs are sorted according to their link gain ratios (i.e., the ratio of the link gain of the SUs toward secondary receiving point and the corresponding link gain toward primary receiving point) and non-supported SUs are removed by using the bisection search algorithm.

The removal criterion proposed in [2], [3], [8] and [9] requires a centralized node to know all the system parameters including instantaneous channel state information (CSI) between all nodes, the target-SINR and maximum transmit power levels for all users. This causes heavy signalling overheads. Furthermore, the complexity of removal algorithms proposed in [2], [3], [8], [9] are of $O(|\mathcal{U}^s|^4)$, $O(|\mathcal{U}^s|^3)$, $O(|\mathcal{U}^s|^3)$, $O(|\mathcal{U}^s|^2 \log |\mathcal{U}^s|)$, respectively, where $|\mathcal{U}^s|$ is the number of SUs (refer to Table I).

In [10], a distributed prioritized power control algorithm is proposed in which the feasibility of target-SINRs for SUs under the constraint of zero-outage for PUs is individually checked by each SU in a distributed manner, where an SU removes itself if that user is unable to reach its target-SINR and/or its

TABLE I
SUMMARY OF RELATED WORK AND COMPARISON WITH OUR PROPOSED APPROACH

Ref.	Type of the Cognitive Network	Type of the Primary Network	Impact of PUs' Transmissions on SUs' Receivers	Solution Category	Assumptions and Required Information
[8]	Multiple secondary transmitter/receiver pairs	Single primary receiver	No	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^4)$	Full knowledge
[9]	Multiple secondary transmitter/receiver pairs	Multiple primary TV receivers and single primary TV station	Yes	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^3)$	Full knowledge
[3]	Multiple secondary transmitter/receiver pairs	Multiple primary transmitter/receiver pairs	No	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^3)$	Full knowledge
[2]	Multiple secondary transmitter/receiver pairs	Multiple primary transmitter/receiver pairs	No	Centralized, complexity $O(\lvert\mathcal{U}^{\mathrm{s}}\rvert^2 \log \lvert\mathcal{U}^{\mathrm{s}}\rvert)$	Full knowledge
[11]	Single-cell served by a secondary BS with antenna array	Single primary receiver and single primary transmitter	Yes	Distributed	Unconstrained PU's transmit power
[6]	Multiple secondary transmitter/receiver pairs	Single primary receiver and multiple primary transmitters	Yes	Distributed	Iterative signal exchanging between SUs to schedule the round robin turns of power updates, updating the current activated and deactivated SUs, broadcasting a warning signals to SUs by primary receiver if the interference temperature constraint limit is violated
[4]	Multiple secondary transmitter/receiver pairs	Single primary receiver	No	Distributed	Each SU is aware of the target-SINR of other users, the path-gain from other interfering users toward its receiver and from that user to the receiver of the other users, iterative message passing between SUs to exchange their instantaneous transmit power levels and dual variables
[10]	Single-cell	Single primary receiver	Yes	Distributed	PUs and SUs connect to a single BS, each SU is aware of total received power temperature and noise level at the BS
Our work	Multi-cell cognitive network	Multi-cell primary network	Yes	Distributed	Iterative broadcasting the ratio of total received power to total received power temperature at the primary receivers to SUs

existence causes outage of a high-priority user. However, [10] focuses on single-cell networks where all SUs and PUs are served by a single base-station. Our current paper considers a sufficiently general system model of multi-cellular networks where a multi-cell secondary radio network coexists with a multi-cell primary network. In [11], a distributed algorithm is introduced to minimize the total transmit power of primary and secondary links using antenna arrays. However, the PUs are allowed to increase their transmit power levels without bounds, which is not practical. In [6], a power and admission control algorithm is proposed to maximize the aggregate throughput for the maximum number of SUs that can be admitted to the network under the constraint of PUs' interference temperature limit. However, this algorithm incurs a significant amount of computation and signalling overhead.

Different from the existing work in the literature, in this paper, we design distributed uplink power control algorithms with reduced signalling overhead and computation complexity for a general system model where there exist multiple primary and secondary transmitter/receivers in a multi-cell environment. The objective is to support the maximal number of SUs with their target-SINRs subject to the constraint of zero-outage ratio for the PUs. In contrast to the works in [2]–[4] where transmit power of primary networks and thus the interference caused by primary networks on secondary networks are assumed fixed, the dynamics of PUs' transmit power is considered in our system model. Furthermore, in contrast to [4], [6], [10], and [11], which consider a CRN with only a single primary receiver, we consider a multi-cell CRN coexisting with a multi-cell primary network as is the case in practice. *Note that, existence of multiple receivers or primary base-stations (BSs) requires us to satisfy the corresponding interference temperature at each BS which makes the problem of power control in underlay CRNs more challenging*. This is due to the fact that, for controlling the transmit power of a given SU, the amount of interference imposed by that SU at different points of primary receivers has to be taken into account.

III. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Model and Notations

Consider an underlay interference-limited cognitive wireless network where a secondary cellular network coexists with a primary cellular network. The secondary network consists of a set of SUs denoted by $\mathcal{U}^{\mathrm{s}}$ which are served by a set of secondary base-stations (SBSs) denoted by $\mathcal{B}^{\mathrm{s}}$. The primary network consists of a set of primary base-stations (PBSs) denoted by $\mathcal{B}^{\mathrm{p}}$ serving the set of PUs denoted by $\mathcal{U}^{\mathrm{p}}$. We assume a fixed base-station assignment in both primary and secondary networks, i.e., each PU or SU is already associated with a fixed BS in the corresponding cells. Let us denote the set of PUs associated to BS $k \in \mathcal{B}^{\mathrm{p}}$ by $\mathcal{U}_k^{\mathrm{p}}$ and the set of SUs associated to BS $k \in \mathcal{B}^{\mathrm{s}}$ by $\mathcal{U}_k^{\mathrm{s}}$. Thus we have $\mathcal{U}^{\mathrm{p}} = \cup_{k\in\mathcal{B}^{\mathrm{p}}}\mathcal{U}_k^{\mathrm{p}}$ and $\mathcal{U}^{\mathrm{s}} = \cup_{k\in\mathcal{B}^{\mathrm{s}}}\mathcal{U}_k^{\mathrm{s}}$. Let us also denote the set of all users by $\mathcal{U} = \mathcal{U}^{\mathrm{p}} \cup \mathcal{U}^{\mathrm{s}}$ and the set of all base stations by $\mathcal{B} = \mathcal{B}^{\mathrm{p}} \cup \mathcal{B}^{\mathrm{s}}$.

Let p_i be the transmit power of user i and $0 \le p_i \le \overline{p}_i$, where $\overline{p}_i$ is the upper limit of the transmit power for user i. Let $\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}$ imply $0 \le p_i \le \overline{p}_i$ for all $i \in \mathcal{U}$. The BS assigned to user i is denoted by $b_i \in \mathcal{B}$ and the path gain from user j to the BS b_i is denoted by $h_{b_i,j}$, and thus the received power of user j at the BS assigned to user i is $p_j h_{b_i,j}$. Noise power at each receiver is assumed to be additive white Gaussian.

The receiver is assumed to be a conventional matched filter. Thus, for a given transmit power vector $\mathbf{p}$, the SINR of user i achieved at its receiver, denoted by γ_i is

$$\gamma_i(\mathbf{p}) \triangleq \frac{p_i h_{b_i,i}}{\sum\limits_{j\in\mathcal{U}, j\neq i} p_j h_{b_i,j} + \sigma_{b_i}^2}, \tag{1}$$

where $\sigma_{b_i}^2$ is the noise power at the receiver of user i. An SINR vector is denoted by $\gamma = [\gamma^{\mathrm{p}}, \gamma^{\mathrm{s}}]$, where γ^{p} and γ^{s} are SINRs of PUs and SUs, respectively.

Let us denote the total received power at a given base station $k \in \mathcal{B}$ by

$$\varphi_k(\mathbf{p}) = \sum_{i\in\mathcal{U}} p_i h_{k,i} + \sigma_k^2. \tag{2}$$

The effective interference for user i is denoted by R_i, and is defined as the ratio of interference caused to each user i to the path gain to its assigned BS, that is

$$R_i(\mathbf{p}) \triangleq \frac{I_i(\mathbf{p})}{h_{b_i,i}}, \tag{3}$$

where $I_i(\mathbf{p}) = \sum\limits_{j\neq i} p_j h_{b_i,j} + \sigma_{b_i}^2$ is the total interference caused to user i at its receiver. Let us also define the effective SINR of user i by

$$\theta_i(\mathbf{p}) = \frac{\gamma_i(\mathbf{p})}{\gamma_i(\mathbf{p}) + 1}, \tag{4}$$

which is the ratio of received power of user i to the total received power plus noise, i.e., $\theta_i(\mathbf{p}) = \frac{p_i h_{b_i,i}}{\varphi_{b_i}(\mathbf{p})}$.

The target-SINR of each user i is denoted by $\widehat{\gamma}_i$, and is usually equivalent to a maximum tolerable bit error rate (BER) below which the user is not satisfied. Correspondingly, the target-effective SINR is $\widehat{\theta}_i = \frac{\widehat{\gamma}_i}{\widehat{\gamma}_i + 1}$. Given a transmit power vector, user i is supported if $\gamma_i(\mathbf{p}) \ge \widehat{\gamma}_i$, or equivalently, if $\theta_i(\mathbf{p}) \ge \widehat{\theta}_i$. Given a transmit power vector $\mathbf{p}$, let us denote the set of supported users by $\mathcal{S}(\mathbf{p}) = \{i \in \mathcal{U} | \theta_i(\mathbf{p}) \ge \widehat{\theta}_i\}$. We also denote the set of supported SUs and PUs by $\mathcal{S}^{\mathrm{p}}(\mathbf{p}) = \mathcal{S}(\mathbf{p}) \cap \mathcal{U}^{\mathrm{p}}$ and $\mathcal{S}^{\mathrm{s}}(\mathbf{p}) = \mathcal{S}(\mathbf{p}) \cap \mathcal{U}^{\mathrm{s}}$, respectively. Their complementary sets are $\mathcal{S}'(\mathbf{p}) = \mathcal{U} - \mathcal{S}(\mathbf{p})$, $\mathcal{S}'^{\mathrm{p}}(\mathbf{p}) = \mathcal{U}^{\mathrm{p}} - \mathcal{S}^{\mathrm{p}}(\mathbf{p})$, and $\mathcal{S}'^{\mathrm{s}}(\mathbf{p}) = \mathcal{U}^{\mathrm{s}} - \mathcal{S}^{\mathrm{s}}(\mathbf{p})$. The cardinality of a given set $\mathcal{A}$ is denoted by $|\mathcal{A}|$. Given a transmit power vector $\mathbf{p}$, let us define the outage-ratio for primary and secondary users denoted by $O^{\mathrm{p}}(\mathbf{p})$ and $O^{\mathrm{s}}(\mathbf{p})$, respectively, as follows:

$$O^{\mathrm{p}}(\mathbf{p}) = \frac{|\mathcal{S}'^{\mathrm{p}}(\mathbf{p})|}{|\mathcal{U}^{\mathrm{p}}|} \quad \text{and} \quad O^{\mathrm{s}}(\mathbf{p}) = \frac{|\mathcal{S}'^{\mathrm{s}}(\mathbf{p})|}{|\mathcal{U}^{\mathrm{s}}|}. \tag{5}$$

In the TPC method proposed in [7], [13], the transmit power for each user i is iteratively set by using

$$p_i(t+1) = \min\left\{\overline{p}_i, f_i^{(\mathrm{T})}(\mathbf{p}(t))\right\}, \tag{6}$$

where

$$f_i^{(\mathrm{T})}(\mathbf{p}(t)) = \widehat{\gamma}_i R_i(\mathbf{p}(t)) \tag{7}$$

in which $R_i(t)$ is the effective interference caused to user i at iteration t and $\overline{p}_i$ is the maximum transmit power constraint. When $p_i(t) \neq 0$, the power-update function in TPC can be rewritten as: $f_i^{(\mathrm{T})}(\mathbf{p}(t)) = \widehat{\theta}_i \varphi(\mathbf{p}(t)) = \frac{\widehat{\theta}_i}{\theta_i(t)} p_i(t) = \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)$, where $\gamma_i(\mathbf{p}(t))$ and $\theta_i(t)$ are the actual SINR and the effective SINR of user i at iteration t, respectively. Convergence to a unique fixed-point[1] is guaranteed for the TPC in both feasible and infeasible systems. However, it suffers from a severe drawback in infeasible systems. Since users employing the TPC rigidly track their target-SINRs, there always exist a few users transmitting at their maximum power without obtaining their target-SINRs, which results in high outage-ratio and high power consumption.

B. Problem Statement

Using matrix notations, the relation between the transmit power vector and the SINR vector can be rewritten as

$$\mathbf{p} = \mathbf{G}.\mathbf{p} + \eta, \tag{8}$$

where the (i,j) component of $\mathbf{G}$ is $G_{i,j} = \frac{h_{b_i,j}\gamma_i}{h_{b_i,i}}$ if $i \neq j$, and $G_{i,j} = 0$ if $i = j$, and the i-th component of η is $\eta_i = \frac{\sigma_{b_i}^2 \gamma_i}{h_{b_i,i}}$.

Definition 1: The target-SINRs of users in a given subset $\mathcal{A} \subseteq \mathcal{U}$ are feasible if there exists a power vector $\mathbf{0} \le \mathbf{p} \le \overline{\mathbf{p}}$ that satisfies the target-SINRs of users in $\mathcal{A}$. In addition, the *system* is *feasible* if the target-SINR vector for all users (i.e., when $\mathcal{A} = \mathcal{U}$) is feasible, otherwise the system is *infeasible*.

It is shown in [5] that the necessary condition for the feasibility of a given SINR vector γ is $\rho(\mathbf{G}) < 1$, where $\rho(\mathbf{G})$ is the spectral radius (maximum eigenvalue) of matrix $\mathbf{G}$. This would be a sufficient condition only if there is no upper limit on transmit power of users (i.e., $\overline{p}_i = \infty$).

[1]In a distributed power control algorithm, each user i updates its transmit power by a power-update function $f_i(\mathbf{p})$, that is, $p_i(t+1) = f_i(\mathbf{p}(t))$, where $\mathbf{p}(t)$ is the transmit power vector at time t. The fixed-point of the power update function, denoted by $\mathbf{p}^*$, is obtained by solving $\mathbf{p}^* = \mathbf{f}(\mathbf{p}^*)$.

Throughout this paper, we suppose that the target-SINRs for the PUs are feasible, i.e., there exists a transmit power vector $\mathbf{0} \leq \mathbf{p} \leq \overline{\mathbf{p}}$ for which $O^{\mathrm{p}}(\mathbf{p}) = 0$. But the target-SINRs for all PUs and SUs together may be infeasible. In an infeasible system, the minimal number of SUs should be gradually removed subject to the constraint that all the PUs are supported with their target-SINRs (zero-outage-ratio for the PUs). We define this as the problem of minimizing the outage-ratio of SUs subject to zero-outage-ratio of PUs as follows:

$$\min_{\mathbf{0}\leq\mathbf{p}\leq\overline{\mathbf{p}}} O^{\mathrm{s}}(\mathbf{p}) \qquad \text{subject to } O^{\mathrm{p}}(\mathbf{p}) = 0, \tag{9}$$

in which the constraint $O^{\mathrm{p}}(\mathbf{p}) = 0$ means $\mathcal{S}^{\mathrm{p}}(\mathbf{p}) = \mathcal{U}^{\mathrm{p}}$, i.e., $\gamma_i(\mathbf{p}) \geq \widehat{\gamma}_i, \forall i \in \mathcal{U}^{\mathrm{p}}$, which is assumed to be feasible.

IV. Proposed Distributed Power Control Algorithm

In this section we present our proposed distributed power control algorithm for uplink power control in CRNs. To avoid outage of a PU due to the existence of SUs, a new upper-limit constraint is imposed on the transmit power levels of SUs in addition to their maximum transmit power constraint $\overline{p}_i$, so that the total interference caused by the SUs to the PUs is kept below a given threshold.

A. Total Received-Power-Temperature at the Primary Base Stations (PBSs)

To guarantee a zero-outage ratio for PUs, the total received power plus noise at each PBS must be below a given threshold, as explained and obtained below. Given the total received power plus noise at the PBS $k \in \mathcal{B}^{\mathrm{p}}$, i.e., $\varphi_k(\mathbf{p})$, the effective target-SINR of user $i \in \mathcal{U}^{\mathrm{p}}$ is reachable if and only if $0 \leq \frac{\widehat{\theta}_i}{h_{b_i,i}} \varphi_{b_i}(\mathbf{p}) \leq \overline{p}_i$. Let $\overline{\varphi}_k$ denote the maximum value of the total received power plus noise at the BS $k \in \mathcal{B}^{\mathrm{p}}$ that can be tolerated by all of its associated PUs. We refer $\overline{\varphi}_k$ as the *total received-power-temperature* for PBS k, which is formally defined and obtained as follows:

$$\overline{\varphi}_k = \max\left\{\varphi \,|\, 0 \leq \frac{\widehat{\theta}_i}{h_{k,i}} \varphi \leq \overline{p}_i,\ \forall i \in \mathcal{U}_k^{\mathrm{p}}\right\} = \min_{i\in\mathcal{U}_k^{\mathrm{p}}}\left\{\frac{\overline{p}_i h_{k,i}}{\widehat{\theta}_i}\right\}. \tag{10}$$

Lemma 1: If the transmit power vector $\mathbf{p}$ satisfies the SINR requirements of all PUs then we have $\varphi_k(\mathbf{p}) \leq \overline{\varphi}_k$, for all $k \in \mathcal{B}^{\mathrm{p}}$, or equivalently, $\max_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\varphi_k(\mathbf{p})}{\overline{\varphi}_k}\right\} \leq 1$.

As can be seen, the total received-power-temperature for each PBS $k \in \mathcal{B}^{\mathrm{p}}$, i.e., $\overline{\varphi}_k$ is a dynamic function of noise level, target-SINRs, channel gains, and maximum transmit power levels for users associated to PBS k. In fact, the values of $\overline{\varphi}_k$ for all $k \in \mathcal{B}^{\mathrm{p}}$ indicate the amount of interference tolerability of PBSs at the primary network in the underlay spectrum access strategy. The value of total received-power-temperature for each PBS is dynamically decreased (increased) as the number of its associated PUs are increased (decreased) and/or channel status of primary network becomes weaker (stronger).

Note that the total received-power-temperature $\overline{\varphi}_k$ is obtained by each PBS based on information pertinent to its own associated users only. Thus each PBS $k \in \mathcal{B}^{\mathrm{p}}$ can compute the value of $\overline{\varphi}_k$ in a distributed manner without requiring to know the channel information of other PUs associated to other base-stations $l \in \mathcal{B}^{\mathrm{p}}$. *If, instead of putting a constraint on total received power, we put a constraint on interference caused by the SUs to the PBS (i.e., the so called interference temperature limit in the literature), then the interference temperature limit for each PBS would depend on the channel gains and the target-SINRs of all PUs including those PUs not associated to that PBS as explained below.*

The maximum value of the total interference caused by the SUs to the BS $k \in \mathcal{B}^{\mathrm{p}}$ that can be tolerated by all of its associated PUs, denoted by $\overline{I}_k$, is formally defined and obtained as follows:

$$\begin{aligned}\overline{I}_k &= \max\left\{I^{\mathrm{s}} \,|\, 0 \leq \frac{\widehat{\theta}_i}{h_{k,i}}\left(I_k^{\mathrm{intp}} + I_k^{\mathrm{extp}} + I^{\mathrm{s}} + \sigma_k^2\right) \leq \overline{p}_i,\ \forall i \in \mathcal{U}_k^{\mathrm{p}}\right\} \\ &= \min_{i\in\mathcal{U}_k^{\mathrm{p}}}\left\{\frac{\overline{p}_i h_{k,i}}{\widehat{\theta}_i}\right\} - \left(I_k^{\mathrm{intp}} + I_k^{\mathrm{extp}} + \sigma_k^2\right),\end{aligned} \tag{11}$$

where I_k^{intp} is the total intra-cell interference (total received power by PUs associated to PBS k) and I_k^{extp} is the total (primary inter-cell) interference caused by those PUs not associated to PBS k. As can be seen, the interference temperature at each PBS k (i.e., $\overline{I}_k$) not only depends on the channel gains and the target-SINR requirements for the associated PUs, but also on the instantaneous value of the total interference caused by those PUs not associated to the PBS k. This is in contrast to the total received-power-temperature which depends on the channel status and the target-SINR requirements of its associated PUs only. For this reason, unlike the traditional literature, we focus on the total received-power-temperature limit as a constraint imposed on the transmit power levels of the SUs. This approach enables us to address the problem of distributed uplink power control in cellular CRNs as will be demonstrated in the following sections.

B. Proposed Distributed Power Control Algorithm

Our proposed TPC with PU-protection algorithm (TPC-PP), as summarized in **Algorithm 1** has the following distributed power-update function:

$$p_i(t+1) = \begin{cases} \min\left\{\overline{p}_i,\ \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}, & \text{for all } i \in \mathcal{U}^{\mathrm{p}} \\ \min\left\{\overline{p}_i,\ \beta(t) p_i(t),\ \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}, & \text{for all } i \in \mathcal{U}^{\mathrm{s}}, \end{cases} \tag{12}$$

where $\beta(t) = \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$.

Algorithm 1 TPC with PU-protection (TPC-PP)

Set $t := 1$, for each user $i \in \mathcal{U}$, initialize the transmit power randomly $p_i(t) = \dot{p}_i$ where $\dot{p}_i \in (0, \bar{p}_i]$ and estimate the CSI values from previous time slot.
repeat
  for all PU $i \in \mathcal{U}^{\mathrm{p}}$ **do**
    Obtain the parameter $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}$ from its own PBS.
    Update the power as

$$p_i(t+1) := \min\left\{\overline{p}_i, \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}.$$

  end for
  Each PBS $k \in \mathcal{B}^{\mathrm{p}}$ multicast the parameter $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ to all SU $i \in \mathcal{U}^{\mathrm{s}}$.
  for each SU $i \in \mathcal{U}^{\mathrm{s}}$ **do**
    Obtain the parameter $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}$ from its own SBS.
    Find $\beta(t) := \min_{k \in \mathcal{B}^{\mathrm{p}}} \left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$.
    Update the power as

$$p_i(t+1) := \min\left\{\overline{p}_i, \beta(t) p_i(t), \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}.$$

  end for
  Update the power vector $\mathbf{p}(t+1) := [p_i(t+1)]_{\forall i \in \mathcal{U}}$.
  Update $t := t + 1$.
until $t = T_{\max}$ **or** convergence to any fixed point.

In TPC-PP, each PU employs the TPC. However, each SU employs the TPC as long as the total received power plus noise power at each PBS k, i.e., $\varphi_k(t)$ is less than the corresponding total received-power-temperature $\overline{\varphi}_k$, otherwise the SU updates its transmit power proportional to $\frac{\overline{\varphi}_k}{\varphi_k(t)} p_i(t)$, which is equivalent to setting the transmit power $p_i(t+1)$ to $\min_{k \in \mathcal{B}^{\mathrm{p}}} \left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\} p_i(t)$. The TPC algorithm is indeed the same as the closed-loop power control algorithm, since the ratio of $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)$ in the TPC algorithm can be viewed as the commands of increasing or decreasing the power in closed-loop power control algorithm, corresponding to $\gamma_i(\mathbf{p}(t)) < \widehat{\gamma}_i$ and $\gamma_i(\mathbf{p}(t)) > \widehat{\gamma}_i$, respectively. Similarly, the term $\frac{\overline{\varphi}_k}{\varphi_k(t)} p_i(t)$ can also be viewed as a power-updating command issued by the PBS to SUs. The proposed power control algorithm for the SUs can be interpreted as follows. Each SU receives two power-updating commands at each iteration, one is unicast from its own receiver, in terms of $\frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))}$, and the other ones are multicast from each PBS to all SUs, in terms of $\frac{\overline{\varphi}_k}{\varphi_k(t)}$.

Indeed, the TPC-PP algorithm uses a mixed-strategy for spectrum access as explained in the following. When there are many PUs with large target-SINR requirements associated to a PBS and/or the corresponding channel gains are poor, the total received-power-temperature for that PBS is set to a very small value [according to (10)]. This corresponds to spectrum overlay strategy. On the other hand, when the number and/or the target-SINR requirements of the PUs actively associated to each PBS is moderate and/or the channel gains are good, the values of total received-power-temperature for the PBSs can be non-zero. These values would indicate the amount of interference tolerability of the primary network in the spectrum underlay strategy. Therefore, by dynamically setting the value of the total received-power-temperature for each PBS in an optimum manner using (10), a mixed-strategy is adopted.

V. Analysis of the Proposed Algorithm

A. Signalling Overhead

In our proposed algorithm, in addition to information that each user requires to update its transmit power using the TPC at each iteration, each SU needs to know the ratio of the total received-power-temperature to the instantaneous total received power plus noise for each PBS, i.e., $\frac{\overline{\varphi}_k}{\varphi_k(t)}$, which is provided by the primary base stations. Thus, in comparison with TPC, the additional signalling overhead that TPC-PP incurs is that it requires each PBS k to iteratively provide the SUs with the value of $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ (via a broadcast message in the control channel). Each PBS k may broadcast the values of $\overline{\varphi}_k$ and $\varphi_k(t)$, individually, or the ratios, i.e., $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ to the SUs. Note that the value of $\overline{\varphi}_k$ needs to be updated by PBS k only when one of its associated PUs, who has the minimum value of $\frac{\overline{p}_i h_{k,i}}{\widehat{\theta}_i}$ among all associated PUs, leaves or enters the system. However, in contrast, the value of $\varphi_k(t)$ needs to be updated at each iteration. Since in practice each SU may cause severe interference only to its nearby PBS, each PBS should inform the values of $\overline{\varphi}_k$ and $\varphi_k(t)$ only to its nearby SUs. Alternatively, each SBS can collect the values of $\overline{\varphi}_k$ and $\varphi_k(t)$ from all the nearby PBSs, and feedback only its minimum ratio, i.e., $\min_{k \in \mathcal{B}^{\mathrm{p}}} \left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$ to its associated SUs. In a practical implementation, the feedback information can be quantized and theses quantized feedback information (bits) can be multicast. This is similar to CSI quantization and feedback commonly used in practice. With this implementation, we can control the feedback overhead and performance trade-off by choosing appropriate number of bits for feedback. These feedback information can also be sent to SBSs by PBSs via a possible wired network between them and then SBSs send these feedback to their own SUs.

One may replace $\frac{\overline{\varphi}_k}{\varphi_k(t)}$ with $\frac{\bar{I}_k}{I_k^{\mathrm{s}}(t)}$ in TPC-PP (12), where $\bar{I}_k$ is the interference temperature given by (11), and $I_k^{\mathrm{s}}(t)$ is the instantaneous value of interference caused by all SUs to the PBS k. In this case, all of the analytical results developed in following sections are still valid. However, note that, the former is preferred to the latter from a practical point of view. This is because in the latter case, in addition to $I_k^{\mathrm{s}}(t)$, each PBS k needs to know the value of $\bar{I}_k$, which is a function of the instantaneous value of the total interference caused by all of those PUs not associated to PBS k, as explained in Section IV-A. Thus, given an instantaneous value of the total received power at the PBS, each PBS requires to compute the total interference caused by its associated PUs and all of non-associated PUs separately. On the other hand, to calculate $\frac{\overline{\varphi}_k}{\varphi_k(t)}$, PBS k can easily obtain its total received-power-temperature by using (10) and the information pertinent to its associated PUs, and also can easily measure the instantaneous value of the total received power at its receiver without requiring to know the individual instantaneous values of interference caused to it by all the PUs (i.e., both the associated and non-associated ones).

B. Existence of Fixed-Point and Its Properties

In this section, we show that there exists at least one fixed-point for our proposed power-update function and all of its fixed-points guarantee zero-outage for the PUs. For a given a target-SINR vector $\gamma=[\gamma^{\mathrm{p}},\gamma^{\mathrm{s}}]$, let $\mathbf{p}^{*\mathrm{T}}(\gamma)$ denote the fixed-point of the TPC power-update function, i.e., $p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\gamma_i R_i(\mathbf{p}^{*\mathrm{T}})\right\}$ for all $i\in\mathcal{U}$, supposing that all PUs and SUs employ the TPC with the target-SINR vector of γ.

Lemma 2: Given a target-SINR vector $\widehat{\gamma}=[\widehat{\gamma}^{\mathrm{p}},\widehat{\gamma}^{\mathrm{s}}]$, the corresponding fixed-point of the TPC power-update function $\mathbf{p}^{*\mathrm{T}}(\gamma)$, and corresponding total received-power-temperature $\overline{\varphi}_k$ for each PBS $k\in\mathcal{B}^{\mathrm{p}}$ obtained from (10), the following observations can be made:

(a) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\leq 1$ (or equivalently, if there exists at least one PBS $k\in\mathcal{B}^{\mathrm{p}}$ for which $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))\geq\overline{\varphi}_k$, which implies that there may exist one PU who is in outage due to TPC), then there exists at least one transmit power vector $\mathbf{p}$ for which the following equalities and inequalities hold:

$$\begin{aligned}&\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p})}\right\}=1\\&0\leq p_i\leq\overline{p}_i,\quad\text{for all } i\in\mathcal{U}\\&\gamma_i(\mathbf{p})=\widehat{\gamma}_i^{\mathrm{p}},\quad\text{for all } i\in\mathcal{U}^{\mathrm{p}}\\&\gamma_i(\mathbf{p})\leq\widehat{\gamma}_i^{\mathrm{s}},\quad\text{for all } i\in\mathcal{U}^{\mathrm{s}}.\end{aligned}\tag{13}$$

(b) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$ (or equivalently, if $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_k$ for all PBS $k\in\mathcal{B}^{\mathrm{p}}$, which implies zero-outage for PUs by TPC), then no transmit power vector exists which satisfies all of the conditions above.

Proof:

(a) Let $l=\arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}$. The target-SINRs of PUs are feasible, i.e., $\gamma'=[\widehat{\gamma}^{\mathrm{p}},0]$ is feasible and thus it is achievable by the TPC and from **Lemma 1** we conclude that $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\gamma'))\leq\overline{\varphi}_k$ for all $k\in\mathcal{B}^{\mathrm{p}}$. Since $\gamma'\leq\widehat{\gamma}$, we have $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\gamma'))\leq\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$ for all $k\in\mathcal{B}^{\mathrm{p}}$. Therefore, if $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\leq 1$, or equivalently, if $\overline{\varphi}_l\leq\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$, we have $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma'))\leq\overline{\varphi}_l\leq\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$. From this and by noting that $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma))$ is a continuous function of γ (because the functions $\varphi_l(\mathbf{p})$ and $\mathbf{p}^{*\mathrm{T}}(\gamma)$ are continuous), we conclude from the Intermediate-Value Theorem [14] that there exists at least one SINR vector γ where $\gamma'\leq\gamma\leq\widehat{\gamma}$ for which $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma))=\overline{\varphi}_l$. Thus there exists a transmit power vector $\mathbf{p}=\mathbf{p}^{*\mathrm{T}}(\gamma)$ that satisfies (13) (because $\varphi_l(\mathbf{p})=\varphi_l(\mathbf{p}^{*\mathrm{T}}(\gamma))=\overline{\varphi}_l$ and $\gamma'\leq\gamma\leq\widehat{\gamma}$ corresponds to two last constraints of (13)).

(b) Let $l=\arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}$. If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, then we have $\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_k$ for all $k\in\mathcal{B}^{\mathrm{p}}$, and thus $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_l$. When $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\overline{\varphi}_l$, if there exists a transmit power vector $\mathbf{p}$ that satisfies (13), we have $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\varphi_l(\mathbf{p})=\overline{\varphi}_l$. From this we conclude that $\gamma_i(\mathbf{p})>\widehat{\gamma}_i$ holds for at least a user $i\in\mathcal{U}$. Because, otherwise, we have $\gamma_i(\mathbf{p})\leq\widehat{\gamma}_i$ for all $i\in\mathcal{U}$ (i.e., $\gamma(\mathbf{p})\leq\widehat{\gamma}$) and hence $\varphi_l(\mathbf{p})\leq\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))$. Since one can show that for any two feasible SINRs, γ_1 and γ_2 and their corresponding power vector $\mathbf{p}_1$ and $\mathbf{p}_2$, if $\gamma_1\leq\gamma_2$ then $\mathbf{p}_1\leq\mathbf{p}_2$, and thus $\varphi(\mathbf{p}_1)\leq\varphi(\mathbf{p}_2)$. This contradicts $\varphi_l(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))<\varphi_l(\mathbf{p})=\overline{\varphi}_l$. This implies that when $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, no transmit power vector $\mathbf{p}$ exists that satisfies (13). ■

Theorem 1: Similar to **Lemma 2**, let $\mathbf{p}^{*\mathrm{T}}$ be the fixed-point of the TPC power-update function when all users employ the TPC algorithm.

(a) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\leq 1$, then any transmit power vector which satisfies the conditions in (13) is a fixed-point of TPC-PP. In this case, the fixed-point of the TPC-PP is not generally unique.

(b) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, then the fixed-point of TPC-PP is unique and the same as $\mathbf{p}^{*\mathrm{T}}$.

Proof: Let $l=\arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}$. We consider the following two cases.

Case (a): From **Lemma 2** we know that, if $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}\leq 1$, then there exists a transmit power vector $\widetilde{\mathbf{p}}$ that satisfies the conditions in (13). To prove that $\widetilde{\mathbf{p}}$ is the fixed-point of TPC-PP, we need to show the following:

$$\widetilde{p}_i=\min\left\{\overline{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{p}}\tag{14}$$

$$\widetilde{p}_i=\min\left\{\overline{p}_i,\frac{\overline{\varphi}_l}{\varphi_l(\widetilde{\mathbf{p}})}\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{s}}.\tag{15}$$

From (13) we conclude that $\widetilde{p}_i=\gamma_i(\widetilde{\mathbf{p}})R_i(\widetilde{\mathbf{p}})=\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})$ and $\widetilde{p}_i\leq\overline{p}_i$ for all $i\in\mathcal{U}^{\mathrm{p}}$ and thus (14) holds. Also, since $\varphi_l(\widetilde{\mathbf{p}})=\overline{\varphi}_l$, we have $\min\{\overline{p}_i,\frac{\overline{\varphi}_l}{\varphi_l(\widetilde{\mathbf{p}})}\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\}=\min\{\overline{p}_i,\widetilde{p}_i,\widehat{\gamma}_i R_i(\mathbf{p})\}$ for all $i\in\mathcal{U}^{\mathrm{s}}$. Hence to prove (15), we only need to show that $\widetilde{p}_i=\min\{\overline{p}_i,\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\}$ holds for all $i\in\mathcal{U}^{\mathrm{s}}$. From (13) we know that $\widetilde{p}_i\leq\overline{p}_i$ and $\widetilde{p}_i=\gamma_i(\widetilde{\mathbf{p}})R_i(\widetilde{\mathbf{p}})\leq\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})$, and consequently, $\widetilde{p}_i=\min\{\overline{p}_i,\widetilde{p}_i,\widehat{\gamma}_i R_i(\widetilde{\mathbf{p}})\}$ holds for all $i\in\mathcal{U}^{\mathrm{s}}$, and hence (15) holds. This completes the proof.

Case (b): If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^{*\mathrm{T}}(\widehat{\gamma}))}\right\}>1$, we first show that $\mathbf{p}^{*\mathrm{T}}$ is a fixed-point of TPC-PP and then show that this fixed-point is unique. To show the former, we need to show that

$$p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\widehat{\gamma}_i R_i\left(\mathbf{p}^{*\mathrm{T}}\right)\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{p}}\tag{16}$$

$$p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\frac{\overline{\varphi}_l}{\varphi_l\left(\mathbf{p}^{*\mathrm{T}}\right)}p_i^{*\mathrm{T}},\widehat{\gamma}_i R_i\left(\mathbf{p}^{*\mathrm{T}}\right)\right\},\ \text{for all } i\in\mathcal{U}^{\mathrm{s}}.\tag{17}$$

Since $\mathbf{p}^{*\mathrm{T}}$ is the fixed-point of the TPC, we have $p_i^{*\mathrm{T}}=\min\left\{\overline{p}_i,\widehat{\gamma}_i R_i(\mathbf{p}^{*\mathrm{T}})\right\}$ for all $i\in\mathcal{U}$ and thus (16) holds. From

$\varphi_l(\mathbf{p}^{*\mathrm{T}}) < \overline{\varphi}_l$, we conclude that $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p})} p_i^{*\mathrm{T}} > p_i^{*\mathrm{T}}$. From this and from $p_i^{*\mathrm{T}} = \min\left\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^{*\mathrm{T}})\right\}$ for all $i \in \mathcal{U}$, (17) is concluded and thus the proof is completed. ■

Theorem 2: Given a fixed-point $\mathbf{p}^*$ for the power-update function of our proposed algorithm, we have $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} \geq 1$ (or equivalently, $\varphi_k(\mathbf{p}^*) \leq \overline{\varphi}_k$ for all $k \in \mathcal{B}^{\mathrm{p}}$). Furthermore,

(a) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} = 1$, then $\mathbf{p}^*$ satisfies the conditions in (13). In this case, the fixed-point for the power-update function of the TPC-PP is generally not unique.

(b) If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} > 1$, then the fixed-point $\mathbf{p}^*$ is the same as the fixed-point of the TPC. In this case, the fixed-point for the power-update function of the TPC-PP is unique.

Proof: If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} < 1$, then we have $p_i^* > \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}{\overline{\varphi}_k}\right\} p_i^*$, and thus the fixed-point constraint (15) cannot hold. Therefore, for any fixed-point we have $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} \geq 1$. We now prove parts (a) and (b).

Part (a): If $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\widehat{\gamma}))}\right\} = 1$, we have $\widehat{\gamma}_i R_i(\mathbf{p}^*) = \frac{\widehat{\theta}_i}{h_{b_i,i}}\varphi_{b_i}(\mathbf{p}^*) \leq \frac{\widehat{\theta}_i}{h_{b_i,i}}\overline{\varphi}_{b_i} \leq \overline{p}_i$, for all $i \in \mathcal{U}^{\mathrm{p}}$, in which the last inequality holds because $\overline{\varphi}_{b_i} = \min_{j\in\mathcal{U}^{\mathrm{p}}_{b_i}}\left\{\frac{\overline{p}_j h_{b_i,j}}{\widehat{\theta}_j}\right\} \leq \frac{\overline{p}_i h_{b_i,i}}{\widehat{\theta}_i}$, for all $i \in \mathcal{U}^{\mathrm{p}}$. Thus $p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\} = \widehat{\gamma}_i R_i(\mathbf{p}^*)$, for all $i \in \mathcal{U}^{\mathrm{p}}$ which implies that $\gamma_i(\mathbf{p}^*) = \widehat{\gamma}_i$ for all $i \in \mathcal{U}^{\mathrm{p}}$. In addition, from $p_i^* = \min\left\{\overline{p}_i, \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^*, \widehat{\gamma}_i R_i(\mathbf{p}^*)\right\}$ for all $i \in \mathcal{U}^{\mathrm{s}}$, we conclude $p_i^* \leq \widehat{\gamma}_i R_i(\mathbf{p}^*)$, or equivalently, $\gamma_i(\mathbf{p}^*) \leq \widehat{\gamma}_i$ for all $i \in \mathcal{U}^{\mathrm{s}}$. Thus $\mathbf{p}^*$ satisfies the constraints in (13).

Part (b): Since $\mathbf{p}^*$ is a fixed-point for our proposed power update function, it satisfies the following fixed-point constraints:

$$p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\}, \text{ for all } i \in \mathcal{U}^{\mathrm{p}}, \tag{18}$$

$$p_i = \min\left\{\overline{p}_i, \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^*, \widehat{\gamma}_i R_i(\mathbf{p}^*)\right\}, \text{ for all } i \in \mathcal{U}^{\mathrm{s}}. \tag{19}$$

To show that $\mathbf{p}^*$ is a fixed-point of the TPC, we need to show that $p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\}$ for all $i \in \mathcal{U}$. From (18) we know that this holds for all $i \in \mathcal{U}^{\mathrm{p}}$ and thus we only need to show this for all $i \in \mathcal{U}^{\mathrm{s}}$. Since $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} > 1$, we conclude that $\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^* > p_i^*$. From this and from (19), we see $p_i^* = \min\left\{\overline{p}_i, \min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} p_i^*, \widehat{\gamma}_i R_i(\mathbf{p}^*)\right\} = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\}$ for all $i \in \mathcal{U}^{\mathrm{s}}$, which completes the proof. ■

In the following Lemma, we derive the key properties of the fixed-points of our proposed power-update function.

Lemma 3: Our proposed algorithm guarantees zero-outage ratio for PUs, i.e., given any fixed-point $\mathbf{p}^*$ of the power-update function of the TPC-PP, we have $O^{\mathrm{p}}(\mathbf{p}^*) = 0$.

Proof: From **Theorem 2** we have $\varphi_k(\mathbf{p}^*) \leq \overline{\varphi}_k$ for all $k \in \mathcal{B}^{\mathrm{p}}$, and thus $\widehat{\gamma}_i R_i(\mathbf{p}^*) = \frac{\widehat{\theta}_i}{h_{b_i,i}}\varphi_{b_i}(\mathbf{p}^*) \leq \frac{\widehat{\theta}_i}{h_{b_i,i}}\overline{\varphi}_{b_i}$ holds for all $i \in \mathcal{U}^{\mathrm{p}}$. Furthermore, since $\overline{\varphi}_{b_i} = \min_{j\in\mathcal{U}^{\mathrm{p}}_{b_i}}\left\{\frac{\overline{p}_j h_{b_i,j}}{\widehat{\theta}_j}\right\} \leq \frac{\overline{p}_i h_{b_i,i}}{\widehat{\theta}_i}$, for all $i \in \mathcal{U}^{\mathrm{p}}$, we have $\frac{\widehat{\theta}_i}{h_{b_i,i}}\overline{\varphi}_{b_i} \leq \overline{p}_i$, from which we conclude $\frac{\widehat{\theta}_i}{h_{b_i,i}}\overline{\varphi}_{b_i} \leq \overline{p}_i$. Thus, for all $i \in \mathcal{U}^{\mathrm{p}}$, we have $\widehat{\gamma}_i R_i(\mathbf{p}^*) \leq \overline{p}_i$ from which we have $p_i^* = \min\{\overline{p}_i, \widehat{\gamma}_i R_i(\mathbf{p}^*)\} = \widehat{\gamma}_i R_i(\mathbf{p}^*)$. This proves that $\gamma_i(\mathbf{p}^*) = \widehat{\gamma}_i$ for all $i \in \mathcal{U}^{\mathrm{p}}$, or equivalently $O^{\mathrm{p}}(\mathbf{p}^*) = 0$. ■

The key properties of our proposed uplink power control algorithm are summarized as follows.

(a) Our proposed algorithm keeps the total received power plus noise at each PBS bellow the threshold given by (10) so that all the PUs attain their target-SINRs. In other words, the TPC-PP guarantees that the existence of the SUs does not cause outage to any PU. When the system is infeasible, all the PUs together with some SUs attain their target-SINRs, and the remaining SUs are unable to obtain their target-SINRs.

(b) When the system is feasible, the fixed-point of TPC-PP is unique and the same as that of the TPC power update function, at which all users attain their target-SINRs consuming minimum aggregate transmit power.

VI. Improved TPC-PP (ITPC-PP)

Although all fixed-points of the power-update function in the proposed TPC-PP algorithm result in zero-outage ratios for PUs, the outage ratios for SUs are not necessarily the same for all fixed-points. Among all possible fixed-points of the TPC-PP algorithm, the fixed-points with minimal outage ratio of SUs would be most desirable. The TPC-PP algorithm may converge to any of its fixed-points, depending of its initial transmit power vector. Now, an important question is how to lead the TPC-PP to converge to a desired fixed-point.

According to TPC-PP power update function in (12), when $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))} < 1$ at any iteration t, where $l = \arg\min_{k\in\mathcal{B}^{\mathrm{p}}}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$, each SU, whether it has high or low path-gain with PBS l, decreases its transmit power in proportion to $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))}$ to make the interference caused by SUs to PBSs lower than the threshold value. However, it is more efficient if the SUs, which cause more interference to PBS l (such SUs have high channel gains to PBS l), decrease their transmit power levels more than the other SUs. Thus, if an SU causes a very low interference to PBS l (such an SU has low channel gain with PBS l), that SU should not decrease its transmit power. This is because, reduction in its power may make it unsupported while not reducing the interference caused to the PBS l significantly. Accordingly, we propose the following improved TPC-PP (ITPC-PP) power update-function:

$$p_i(t+1) = \begin{cases} \min\left\{\overline{p}_i, \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}, & \text{for all } i \in \mathcal{U}^{\mathrm{p}} \\ \min\left\{\overline{p}_i, \beta_i(t) p_i(t), \frac{\widehat{\gamma}_i}{\gamma_i(\mathbf{p}(t))} p_i(t)\right\}, & \text{for all } i \in \mathcal{U}^{\mathrm{s}} \end{cases} \tag{20}$$

where

$$\beta_i(t)=\begin{cases}\beta(t), & \text{if } \beta(t)\geq 1\\ \beta(t)\left(1+|\overline{\varphi}_l-\varphi_l(\mathbf{p}(t))|\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}}\right), & \text{if } \beta(t)<1\end{cases} \tag{21}$$

In (21), $\beta(t) = \min_{k\in\mathcal{B}^p}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}(t))}\right\}$ and $l = \arg\min_{k\in\mathcal{B}^p}\{\overline{\varphi}_k/\varphi_k(\mathbf{p}(t))\}$.

From the viewpoint of signalling overhead, in ITPC-PP, in addition to the information required in TPC-PP, each SU needs to know (estimate) its channel gain with PBS l. In fact, the only difference between ITPC-PP and TPC-PP is that when $\min_{k\in\mathcal{B}^p}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\tilde{\gamma}))}\right\} < 1$, ITPC-PP causes each SU i to decrease its transmit power level in proportion to $\beta(t)\left(1+|\overline{\varphi}_l-\varphi_l(\mathbf{p}(t))|\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}}\right)$. On the other hand, in TPC-PP, all SUs decrease their transmit powers in proportion to $\beta(t)$. If the effective interference experienced by a given SU i at PBS l is lower than that of SU j, i.e., if $\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}} < \frac{\varphi_l(\mathbf{p}(t))-p_j(t)h_{l,j}}{h_{l,j}}$, the channel gain of SU i toward PBS l is better than that of SU j, and consequently, SU i causes more interference toward PBS l as compared to SU j. In this case, if $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))} < 1$, SU i should reduce its transmit power more than SU j. This is done by adjusting $\beta_i(t)$ according to (21), because $\frac{\varphi_l(\mathbf{p}(t))-p_i(t)h_{l,i}}{h_{l,i}} < \frac{\varphi_l(\mathbf{p}(t))-p_j(t)h_{l,j}}{h_{l,j}}$ results in $\beta_i(t) < \beta_j(t)$ which causes SU i to decrease its power more in comparison with SU j. Therefore, with the proposed ITPC-PP, the SUs close to PBS l reduce their transmit power more as compared to SUs far from PBS l.

Theorem 3: Any fixed-point $\mathbf{p}^*$ for the TPC-PP power-update function (12) is also a fixed-point for the ITPC-PP power-update function (20).

Proof: Given a fixed-point $\mathbf{p}^*$ of the TPC-PP power-update function, from **Theorem 2**, we know that $\min_{k\in\mathcal{B}^p}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*)}\right\} \geq 1$. Thus, $\mathbf{p}^*$ is also a fixed-point for the ITPC-PP power-update function in (20), because when $\min_{k\in\mathcal{B}^p}\left\{\frac{\overline{\varphi}_k}{\varphi_k(\mathbf{p}^*(\tilde{\gamma}))}\right\}\geq 1$, we have $f_i^{\mathrm{ITPC-PP}}(\mathbf{p}^*) = f_i^{\mathrm{TPC-PP}}(\mathbf{p}^*)$ where $f_i^{\mathrm{TPC-PP}}(\mathbf{p})$ and $f_i^{\mathrm{ITPC-PP}}(\mathbf{p})$ are the power-update functions of TPC-PP and ITPC-PP, respectively. ■

Note that although any fixed-point of TPC-PP is also a fixed-point of ITPC-PP, when $\frac{\overline{\varphi}_l}{\varphi_l(\mathbf{p}(t))} < 1$, since ITPC-PP causes the SUs with high channel gains toward PBS l decrease their transmit power levels more aggressively, a fixed-point with improved outage ratio for SUs is eventually reached for ITPC-PP, while zero-outage ratio for PUs is still guaranteed. This will be demonstrated via the simulation results presented in Section VIII-B.

VII. Distributed Power Allocation Under Channel Uncertainties

The distributed power control approaches discussed in preceding sections are based upon the assumption that perfect channel information is known to the receivers, which may not be the case in practice. Therefore, in the following, we modify the power update equations for the TCP-PP algorithm considering uncertainties in channel gains. For this, we approximate the channel gain variations using ellipsoidal uncertainty sets [15]. We refer to the modified algorithm as robust TCP-PP (RTCP-PP).

A. Uncertainty Sets

Let us define the normalized channel gain of user i assigned to BS b_i as follows:

$$F_{i,j} = \begin{cases}\frac{h_{b_i,j}}{h_{b_i,i}}, & \text{if } i\neq j\\ 0, & \text{otherwise.}\end{cases} \tag{22}$$

We model the imperfect channel gains as

$$\tilde{F}_{i,j} = F_{i,j}+\Delta F_{i,j},\ \forall i,j\in\mathcal{U} \tag{23}$$

$$\tilde{h}_{k,i} = h_{k,i}+\Delta h_{k,i},\ \forall k\in\mathcal{B}, i\in\mathcal{U}, \tag{24}$$

where $\tilde{F}_{i,j}$ and $\tilde{h}_{k,i}$ are the actual (or uncertain) value obtained from nominal (or estimated) gains and corresponding perturbation part, e.g., $\Delta F_{i,j}$ and $\Delta h_{k,i}$, respectively. Without loss of generality, let $\mathbf{F}_i = [F_{i,j}]_{\forall j\in\mathcal{U}}$ and $\mathbf{H}_k = [h_{k,i}]_{\forall i\in\mathcal{U}}$ denote the normalized channel gain vector for user $i\in\mathcal{U}$ and the channel gain vector for BS $k\in\mathcal{B}$, respectively. Likewise, $\Delta\mathbf{F}_i$ and $\Delta\mathbf{H}_k$ represent the corresponding perturbation vectors. We approximate the uncertainties in the vector $\mathbf{F}_i$ and $\mathbf{H}_k$ due to fluctuations of the wireless link gains by ellipsoids. Let ξ_{F_i} and ξ_{H_k} represent the maximal deviation of each entries in $\mathbf{F}_i$ and $\mathbf{H}_k$. Under ellipsoidal approximation, the corresponding uncertainty sets $\widetilde{\mathcal{F}}_i$ and $\widetilde{\mathcal{H}}_k$ for $\mathbf{F}_i$ and $\mathbf{H}_k$, respectively, can be written as

$$\widetilde{\mathcal{F}}_i = \left\{\mathbf{F}_i+\Delta\mathbf{F}_i : \sum_{j\neq i}|\Delta F_{i,j}|^2\leq\xi_{F_i}^2\right\},\ \forall i\in\mathcal{U} \tag{25}$$

$$\widetilde{\mathcal{H}}_k = \left\{\mathbf{H}_k+\Delta\mathbf{H}_k : \sum_{i\in\mathcal{U}}|\Delta h_{k,i}|^2\leq\xi_{H_k}^2\right\},\ \forall k\in\mathcal{B}. \tag{26}$$

Using the uncertainty set $\widetilde{\mathcal{F}}_i$ the SINR expression in (1) can be equivalently written as follows [16], [17]:

$$\breve{\gamma}_i(\mathbf{p}) \triangleq \frac{p_i}{\sum_{j\in\mathcal{U},j\neq i} p_j(F_{i,j}+\Delta F_{i,j})+\tilde{\sigma}_{b_i}^2}, \tag{27}$$

where $\tilde{\sigma}_{b_i}^2 = \frac{\sigma_{b_i^2}}{h_{b_i,i}}$. Likewise, the total interference power at BS $k\in\mathcal{B}$ given by (2) can be written as

$$\breve{\varphi}_k(\mathbf{p}) = \sum_{i\in\mathcal{U}} p_i(h_{k,i}+\Delta h_{k,i})+\sigma_k^2. \tag{28}$$

Utilizing the Cauchy-Schwarz inequality [18], we obtain,

$$\sum_{j\in\mathcal{U},j\neq i} p_j\Delta F_{i,j} \leq \sqrt{\sum_{j\in\mathcal{U},j\neq i}|p_j|^2\sum_{j\in\mathcal{U},j\neq i}|\Delta F_{i,j}|^2}$$
$$\leq\xi_{F_i}\sqrt{\sum_{j\in\mathcal{U},j\neq i}p_j^2}. \tag{29}$$

Similarly,

$$\sum_{i\in\mathcal{U}} p_i \Delta H_{k,i} \leq \xi_{H_k} \sqrt{\sum_{i\in\mathcal{U}} p_i^2}. \tag{30}$$

From (29) and (30), we can rewrite (27) and (28) under channel uncertainties as follows:

$$\widetilde{\gamma}_i(\mathbf{p}) = \frac{p_i}{\sum\limits_{j\in\mathcal{U},j\neq i} p_j F_{i,j} + \xi_{F_i}\sqrt{\sum\limits_{j\in\mathcal{U},j\neq i} p_j^2} + \tilde{\sigma}_{b_i}^2} \tag{31}$$

$$\widetilde{\varphi}_k(\mathbf{p}) = \sum_{i\in\mathcal{U}} p_i h_{k,i} + \xi_{H_k}\sqrt{\sum_{i\in\mathcal{U}} p_i^2} + \sigma_k^2. \tag{32}$$

B. Iterative Power Update Under Channel Uncertainty

For any time instance t, let us define the parameter $\mathfrak{Q}(t) = \sqrt{\sum_{i\in\mathcal{U}} p_i^2(t)}$. Then the power update functions are given by (33), (See equation at the bottom of the page) where $\widetilde{\beta}(t) = \min_{k\in\mathcal{B}^{\mathrm{p}}} \left\{ \frac{\overline{\varphi}_k}{\widetilde{\varphi}_k(\mathbf{p}(t))} \right\}$.

Note that the power update functions in (33) for RTCP-PP are similar to those for the TCP-PP algorithm with the modified SINR expression $\widetilde{\gamma}_i(\mathbf{p})$ and received-power-temperature ratio $\widetilde{\beta}(t)$ as well as an additive term. This additive term $\xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)}$ for $\forall i \in \mathcal{U}$ is referred to as *protection function* [15], [19] against uncertainties. The users broadcast their transmit powers every time slot, from which the BSs independently calculate the parameter $\mathfrak{Q}(t)$ and multicast this to the corresponding users. Hence the users can update the power independently similar to **Algorithm 1**. If the uncertainty parameters ξ_{F_i}, ξ_{H_k} for $\forall i,k$ become zero, RTCP-PP reduces to the TPC-PP algorithm, i.e., no channel uncertainty is taken into consideration.

The RTCP-PP algorithm is robust against channel uncertainties since it considers the uncertainties ahead of time, which are deterministically calculated from the realizations of the uncertain parameters to certain extent (i.e., a bounded error region). The algorithm therefore becomes robust to the channel uncertainties at the cost of some performance degradation (which will be explained in the Section VIII-C). As the bounds (i.e., ξ_{F_i}, ξ_{H_k}) become higher, the system becomes more robust against channel uncertainties. However, larger values of the bounds may affect the performance (e.g., achievable SINR, outage ratio etc.) significantly.

VIII. Simulation Results

We present numerical results to illustrate the performances of our proposed TPC-PP, ITPC-PP, and RTPC-PP algorithms and compare them with that of the TPC algorithm. The uplink channel gain from each user i to each BS k is given by $0.1d_{k,i}^{-3}$ where $d_{k,i}$ is the distance between user i and BS k. The upper bound on the transmit power for all users is 1 Watt. We first consider a single snapshot of locations of users and BSs in the network to obtain insight into how TPC-PP works in comparison with the TPC, and then proceed to different snapshots of users' and BSs' locations to verify that the results do not depend on specific user-locations. In Sections VIII-A and VIII-B we show the numerical results assuming that perfect CSI is available to the receivers. Section VIII-C demonstrates the performance results under channel uncertainty.

A. Single Snapshot Scenario

Let us consider a network where 6 PUs and 6 SUs are fixed and served by two PBSs and two SBSs, respectively, in an area of 1000 m × 1000 m, as illustrated in Fig. 1. In this network, each primary (secondary) BS serves 3 primary (secondary) users. For simplicity, suppose that the target SINRs for all PUs and SUs is 0.20. The simulation results for two cases in which users iteratively update their transmit power levels using TPC or TPC-PP, respectively, are shown in Figs. 2–4. Fig. 2 illustrates the total received power plus noise for each PBS normalized by its corresponding total received-power-temperature, i.e., $\frac{\varphi_k(\mathbf{p})}{\overline{\varphi}_k}$, $k \in \mathcal{B}^{\mathrm{p}}$, versus iteration number, for TPC and TPC-PP. The transmit power levels and SINRs for SUs and PUs are shown in Fig. 3(a) and (b) and Figs. 3(a)–4(b) for TPC and TPC-PP, respectively. When the TPC is employed, the total received power at PBS 1 exceeds its maximum received power-temperature, and thus zero-outage ratio for PUs connecting to this PBS is not guaranteed, as shown in Figs. 2 and 3(b). However, when TPC-PP is employed, the total received power at each PBS does not exceed its corresponding maximum received-power-temperature (see Fig. 2), which guarantees zero-outage ratio for PUs (see Fig. 4(b)). Furthermore, by employing TPC-PP, at the equilibrium, we have $\varphi_1(\mathbf{p}) = \overline{\varphi}_1$ for PBS 1, as it was shown in **Theorem 2**. More specifically, as seen in Figs. 4(a) and 3(b), by employing TPC, 4 users including two PUs and two SUs are unable to reach their target-SINRs, whereas by employing TPC-PP, only 3 users are unsupported and these users do not include any PU (see Fig. 4(a) and (b)). This demonstrates that TPC-PP not only guarantees zero-outage ratio for PUs (as shown in **Lemma 3**), but also improves the number of supported SUs.

$$p_i(t+1) = \begin{cases} \min\left\{ \overline{p}_i, \frac{\widehat{\gamma}_i}{\widetilde{\gamma}_i(\mathbf{p}(t))}\left(p_i(t) + \xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)} \right) \right\}, & \forall i \in \mathcal{U}^{\mathrm{p}} \\ \min\left\{ \overline{p}_i, \widetilde{\beta}(t)\left(p_i(t) + \xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)} \right), \frac{\widehat{\gamma}_i}{\widetilde{\gamma}_i(\mathbf{p}(t))}\left(p_i(t) + \xi_{F_i}\sqrt{\mathfrak{Q}^2(t) - p_i^2(t)} \right) \right\}, & \forall i \in \mathcal{U}^{\mathrm{s}}. \end{cases} \tag{33}$$

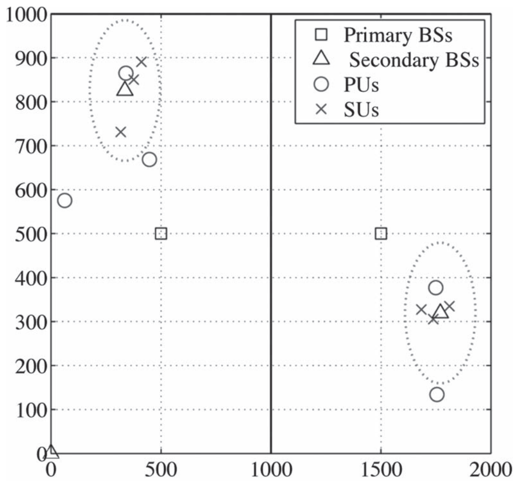


Fig. 1. Network topology and the placement of users and base stations.

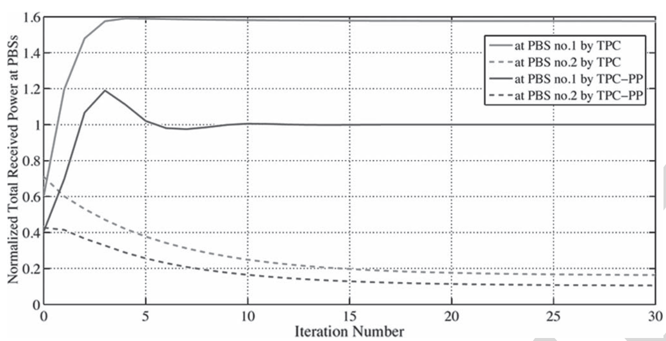


Fig. 2. Normalized total received power plus noise for each PBS versus iteration number, defined as $\frac{\varphi_k(\mathbf{p}(t))}{\overline{\varphi}_k}$ for PBS k, where $k = \{1, 2\}$, for the TPC and TPC-PP algorithms.

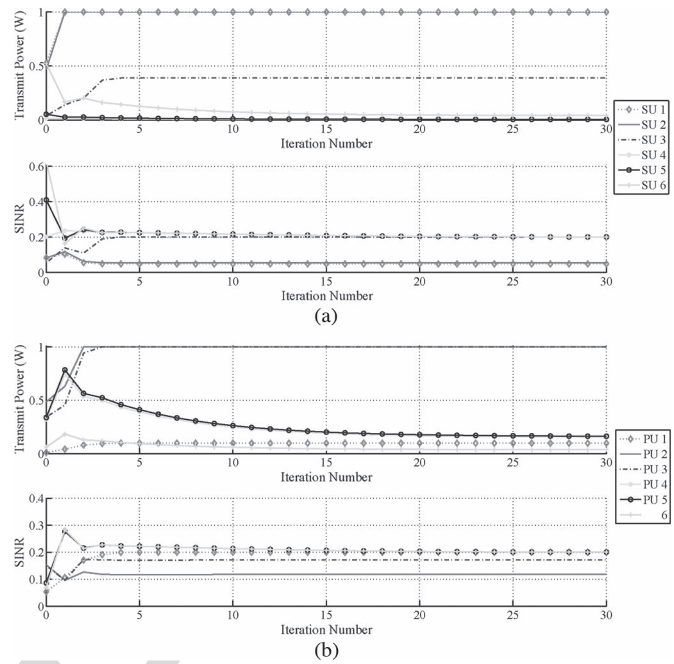


Fig. 3. Transmit power and SINR versus iteration for the TPC algorithm: (a) for SUs, (b) for PUs.

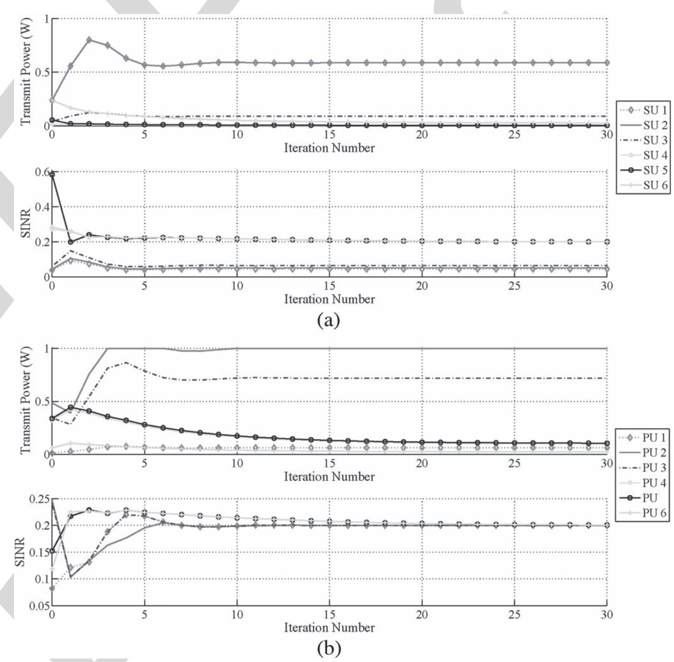


Fig. 4. Transmit power and SINR versus iteration for the TPC-PP algorithm: (a) for SUs, (b) for PUs.

B. Different Snapshots

Now, we compare the performances of TPC-PP, ITPC-PP, and TPC for different snapshots of users' locations and for different values of target-SINRs. For benchmarking purpose, we also compare the performance of our proposed distributed algorithms with a centralized approach called the link gain ratio-based algorithm (LGR) proposed in [2].

Unlike the existing centralized joint power and admission control algorithms, LGR predetermines the admission order of secondary users based on a the link gain ratio metric defined as $\min_{k\in\mathcal{B}^p}\{\bar{I}_k \frac{h_{b_i,i}}{h_{k,i}}\}$, where $\bar{I}_k$ is the interference temperature. In [2], $\bar{I}_k$ is assumed to be fixed, whereas in our case it is dynamic as discussed in Section IV-A and it is given by (11). For this reason, for simulating LGR, we use the total received-power-temperature instead of the interference temperature, i.e., the LGR metric $\min_{k\in\mathcal{B}^p}\{\overline{\varphi}_k \frac{h_{b_i,i}}{h_{k,i}}\}$ is adopted. Using bisection search, the LGR algorithm admits as many secondary users as possible with highest LGRs (or equivalently, removes as few secondary users as possible with lowest LGRs) so that the target-SINRs for all the PUs and the remaining SUs get feasible. The complexity of LGR algorithm is of $O(|\mathcal{U}_s|^2 \log |\mathcal{U}_s|)$ [2], where $|\mathcal{U}_s|$ is the total number of SUs. A drawback of LGR algorithm is that it does not consider different values of the target-SINRs in the admission of the secondary users. Different target-SINRs are possible in networks where different applications are used by different users.

Let us consider a primary network with 3×3 cells where each primary cell covers an area of 1000 m $\times$ 1000 m. Each primary (secondary) user is associated with only one primary (secondary) BS. Each PBS is located at the centre of its corresponding cell and serves 5 PUs. Within this primary network of 3×3 cells, we consider a secondary radio network under two scenarios, namely, with *small cells* (i.e., cells with small transmission radius) and with *large cells* (i.e., cells with larger

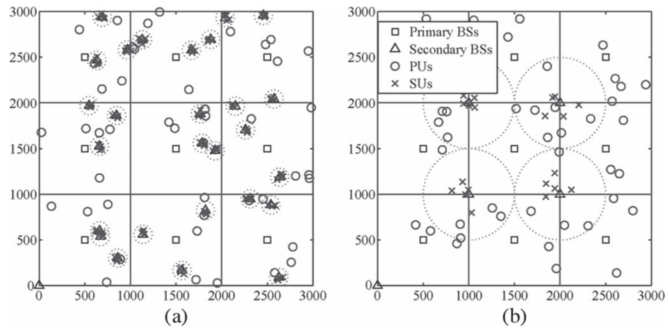


Fig. 5. An example of network topology for a primary network with 3×3 cells with 5 PUs per primary cell, which coexists with a secondary network with small cells [Fig. 5(a)] and large cells [Fig. 5(b)]. The network in Fig. 5(a) includes 3 secondary BSs within each primary cell and 5 SUs per each secondary BS, and the network in Fig. 5(b) includes 4 secondary BSs and 5 SUs per each secondary BS.

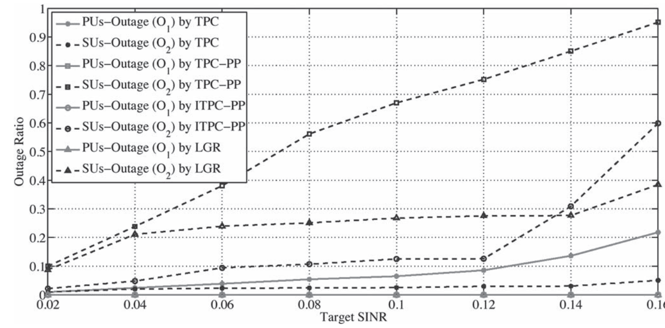


Fig. 6. Average Outage ratios for PUs (O_1) and for SUs (O_2) versus different values of target-SINRs for TPC, TPC-PP, ITPC-PP, and LGR in a small cell CRN. Note that $O_1 = 0$ for TPC-PP, ITPC-PP, and LGR.

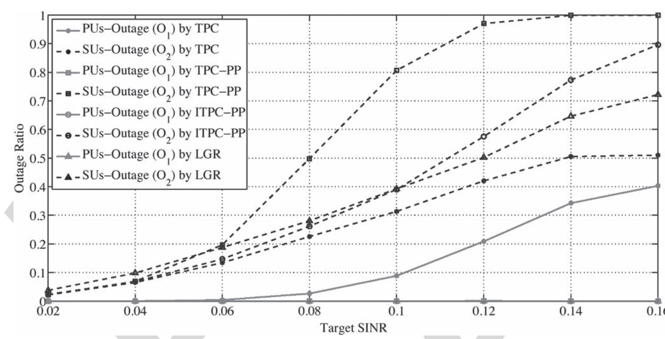


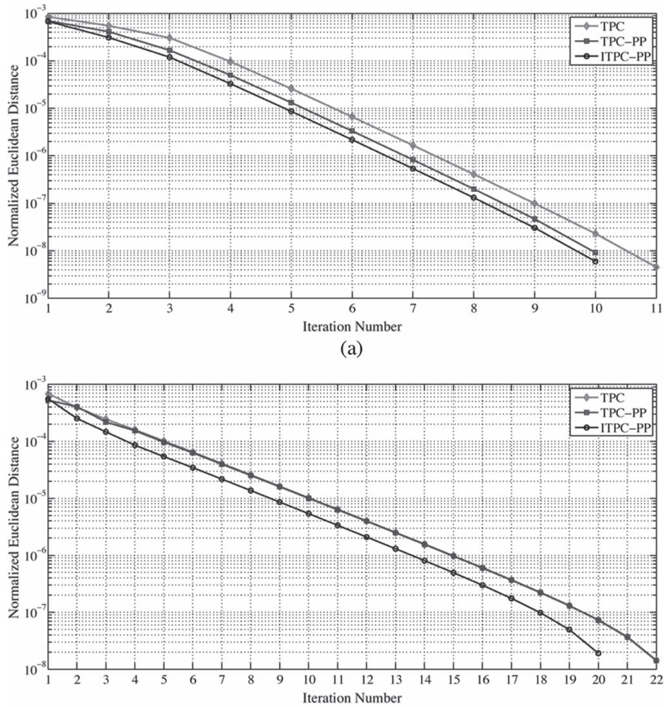
Fig. 7. Average Outage ratios for PUs (O_1) and for SUs (O_2) versus different values of target-SINRs for TPC, TPC-PP, ITPC-PP, and LGR in a large cell CRN. Note that $O_1 = 0$ for TPC-PP, ITPC-PP, and LGR.

transmission radius). The target-SINRs are considered to be the same for all users, ranging from 0.02 to 0.16 with step size of 0.02. Since for values of target-SINR higher than 0.16, the target-SINRs for PUs become infeasible, we use 0.16 as the upper limit of the target-SINR of users. For each target-SINR, we average the corresponding values of outage ratios for the PUs and SUs for TPC, TPC-PP, ITPC-PP, and LGR algorithms for 1000 independent snapshots for a uniform distribution of BSs and users' locations. The initial transmit power for each user is uniformly set from the interval [0,1] for each snapshot.

1) Secondary Radio Network With Small Cells: At each primary cell, 3 secondary BSs are uniformly located, each of which serves 8 SUs uniformly located at a radius of 200 m around it. Thus, the entire network consists of 9 PBSs, 45 PUs, 27 secondary BSs, and 135 SUs. An example of such a network setting is shown in Fig. 5(a). Fig. 6 shows the average outage ratio versus target-SINR, for TPC, TPC-PP, ITPC-PP, and LGR, over 1000 independent snapshots of uniformly distributed locations of users and secondary BSs. Note that both TPC-PP and ITPC-PP outperform TPC with respect to the capability of guaranteeing a zero-outage ratio for PUs at the cost of increased outage ratio for SUs. Moreover, when the total interference caused by SUs to PBS is higher than the threshold, in ITPC-PP, SUs, which cause more interference to PBSs, reduce their transmit power levels. This results in a lower outage ratio for the SUs as compared to the TPC-PP in which all SUs reduce their transmit power levels. For instance, with target-SINR of 0.12, by using TPC-PP and ITPC-PP, the outage ratio for the SUs is 0.75, and 0.12, respectively. When the target-SINR is increased, for example, with target-SINR of 0.16, more SUs have to be removed to keep the total received power below the total received-power-temperature (which is low due to high-SINR requirement by PUs). In other words, the lower and higher values of the SINR requirements for the PUs result in higher and lower values of total received-power-temperature at PBSs, respectively, which correspond to the underlay and the overlay spectrum access strategies used by TPC-PP and ITPC-PP. Thus TPC-PP and ITPC-PP adaptively use a mixed-strategy for spectrum access. Furthermore, ITPC-PP and TPC-PP result in zero-outage ratio for PUs as in LGR, and ITPC-PP follows the outage ratio for SUs as obtained by the centralized LGR algorithm. Specifically, for lower values of target-SINRs, the ITPC-PP algorithm is superior to the LGR algorithm, but it is inferior for higher values of target-SINRs. This shows that our proposed distributed algorithm has comparable performance to that of the centralized LGR algorithm, however at a much lower complexity.

2) Secondary Radio Network With Large Cells: Now consider a CRN with 4 large-cells each of which serves 5 SUs uniformly located at a radius of 1000 m around it within the coverage area of the 3×3 cells primary network. Thus, the entire network consists of 9 PBSs, 45 PUs, 4 secondary BSs, and 20 SUs. An example of such a network setting is shown in Fig. 7(b). Fig. 7 shows the average outage ratio versus target-SINR, for TPC, TPC-PP, and ITPC-PP, over 1000 independent snapshots of uniformly distributed locations of users. Similar to the secondary network setting with small cells, ITPC-PP outperforms TPC-PP in terms of outage ratio for SUs, and both outperform TPC with respect to the capability of guaranteeing a zero-outage ratio for PUs at the cost of increased outage ratio for SUs. Also, the ITPC-PP algorithm follows the outage ratios of PUs and SUs obtained by the centralized LGR algorithm.

In Fig. 8, we illustrate the average rate of convergence of TPC, TPC-PP and ITPC-PP algorithms for both the small cell and large cell scenarios explained above. The rate of convergence $\tau(t)$ at iteration t is measured as normalized Euclidean distance of transmit power, e.g., $\tau(t) = \frac{\|\mathbf{p}(t)-\mathbf{p}^*\|_2}{\|\mathbf{p}(0)-\mathbf{p}^*\|_2}$, where $\mathbf{p}(0)$ is the initial transmit power vector, $\mathbf{p}(t)$ is the transmit power vector at iteration t, $\mathbf{p}^*$ is the fixed-point [corresponding to



(a)

(b)

Fig. 8. Rate of convergence versus iteration for the TPC, TPC-PP and ITPC-PP algorithms: (a) secondary network with small cells and (b) secondary network with large cells. The rate of convergence is measures as normalized Euclidean distance of transmit power, which is given by $\frac{\|\mathbf{p}(t)-\mathbf{p}^*\|_2}{\|\mathbf{p}(0)-\mathbf{p}^*\|_2}$.

the initial transmit power vector $\mathbf{p}(0)$] to which the algorithm converges, and $\|.\|_2$ denote the Euclidean norm. We select the target-SINR for each PU and SU randomly from the set of {0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14} and average the results over 1000 independent simulation realizations. The rest of the simulation parameters are the same as those mentioned for small cell and large cell scenarios. As can be seen from Fig. 8, the rate of convergence of our proposed algorithms is improved in comparison with that of the TPC, which is known to be a fast convergent distributed power control algorithm. In particular, TPC-PP outperforms TPC for small cell scenarios and provides similar convergence rate for large cell scenarios. ITPC-PP outperforms TPC and TPC-PP both for small cell and large cell scenarios.

C. Performance Under Channel Uncertainty

In the following, we observe the performance of our proposed algorithms considering the uncertainty in channel gains. We measure the uncertainty in channel gains as percentages and assume the similar uncertainty bounds in the CSI values for all users. For example, uncertainty bound $\xi = \xi_{F_i} = \xi_{H_k} = 0.02$ means that estimation error in the CSI values $\mathbf{F}_i$ and $\mathbf{H}_k$, $\forall i,k$ is not more than 2% of their nominal values. The numerical results are averaged over 200 independent network realizations. The target-SINRs for all PUs and SUs are set to 0.10 and the rest of the simulation parameters are same as those mentioned in Section VIII-A. Different power control schemes used in the simulations to observe the performance under channel uncertainties are summarized in Table II.

TABLE II
POWER CONTROL SCHEMES USED FOR PERFORMANCE COMPARISON UNDER CHANNEL UNCERTAINTY

Scheme	Update Expression	Ref. Figure(s)
1. TPC (perfect CSI)	(6)	Fig. 11
2. TPC-PP (perfect CSI)	(12)	Fig. 11
3. TPC (uncertain CSI)	(6) with $\gamma_i(\mathbf{p}(t))$ is replaced by $\widetilde{\gamma}_i(\mathbf{p}(t))$	Figs. 9-11
4. TPC-PP (uncertain CSI)	(12) with $\gamma_i(\mathbf{p}(t))$ and $\beta(t)$ are replaced by $\widetilde{\gamma}_i(\mathbf{p}(t))$ and $\widetilde{\beta}(t)$, respectively	Figs. 9-11
5. RTPC	(34)	Figs. 9-11
6. RTPC-PP	(33)	Figs. 9-11

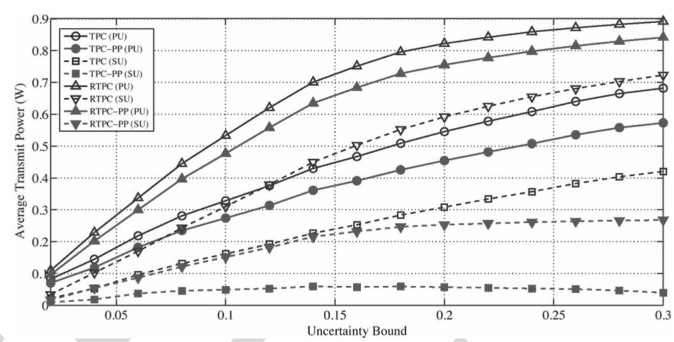


Fig. 9. Average transmit power versus uncertainty bound in TPC, TPC-PP, RTPC, and RTPC-PP algorithm under imperfect CSI.

Note that under channel uncertainties, the TPC power update expression (referred to as robust TPC [RTPC]) for all $i \in \mathcal{U}$ is given by

$$p_i(t+1) = \min\left\{\overline{p}_i, \frac{\widehat{\gamma}_i}{\widetilde{\gamma}_i(\mathbf{p}(t))}\left(p_i(t) + \xi_{F_i}\sqrt{\Omega^2(t) - p_i^2(t)}\right)\right\}. \quad (34)$$

In Fig. 9, considering uncertainty in CSI, we plot the average transmit power[2] for the PUs and SUs with uncertainty bound for both the RTPC and RTPC-PP algorithms. Note that with uncertain CSI values, as mentioned in third and fourth rows of Table II, the parameters $\gamma_i(\mathbf{p}(t))$ and $\beta(t)$ in the power update expression of TPC and TPC-PP algorithm will be replaced with $\widetilde{\gamma}_i(\mathbf{p}(t))$ and $\widetilde{\beta}(t)$, respectively. When the uncertainty bound increases, the PUs increase the transmit power to achieve target-SINR.

Under channel uncertainties, higher uncertainty bounds imply higher fluctuations in CSI values and hence the users require higher transmit powers to overcome the impact of channel uncertainty. Although both the PUs and SUs increase the power in RTPC (TPC) algorithm, RTPC-PP (TPC-PP) prevents the SUs from increasing the power using the parameter $\widetilde{\beta}$ and hence transmit power of SUs are less in RTPC-PP (TPC-PP) compared to RTPC (TPC) which also minimizes the effect of interference from SUs. Another interesting observation is that as the uncertainty bound keeps increasing, the total power approaches to the upper limit $\overline{p}_i$.

[2]The average transmit powers for the PUs and SUs are given by $\frac{\sum p_i}{|\mathcal{U}^{\mathrm{P}}|}$ and $\frac{\sum p_i}{|\mathcal{U}^{\mathrm{s}}|}$, respectively. Similarly, the average SINRs for PUs and SUs are calculated as $\frac{\sum \gamma_i(\mathbf{p})}{|\mathcal{U}^{\mathrm{P}}|}$ and $\frac{\sum \gamma_i(\mathbf{p})}{|\mathcal{U}^{\mathrm{s}}|)}$.

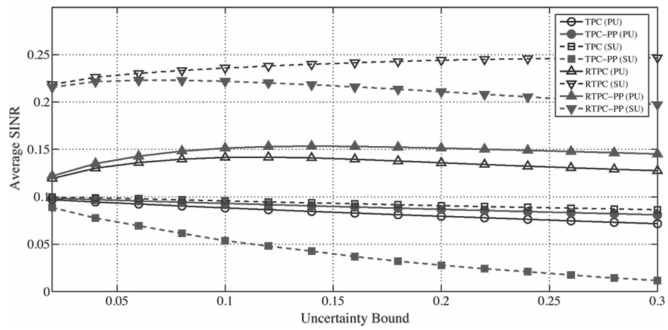


Fig. 10. Average SINR versus uncertainty bound in TPC, TPC-PP, RTPC, and RTPC-PP algorithm under imperfect CSI.

The impact of higher transmit power on users' achievable SINR under channel uncertainties is shown in Fig. 10. As we have seen in Fig. 9, users need to increase their transmit powers to achieve target-SINR. However, higher transmit powers cause more interference at the BSs and hence the SINR decreases at higher uncertainty bounds. Besides, since TPC-PP[3] and TPC do not consider any channel uncertainties, as the uncertainty bound increases, SINR of TPC-PP (TPC) decreases significantly compared to RTPC-PP (RTPC). This is due to the fact that RTPC-PP (RTPC) provides robustness against uncertainties by means of protection function and the users update their power accordingly to achieve target-SINR. Recall that, the RTPC algorithm does not consider interference temperature at the PBSs. Hence, the SUs increase their transmit powers to overcome channel uncertainties, which causes severe interference to PBSs and the SINR for the PUs decreases significantly compared to the proposed RTPC-PP algorithm. In addition, as the transmit power of all the users reaches to its maximum limit, increasing the uncertainty bounds reduces SINR. Note that the SINR expression in (31) using protection function can be written as $\widetilde{\gamma}_i(\mathbf{p}) = \frac{p_i}{\sum\limits_{j\in\mathcal{U},j\neq i} p_j F_{i,j} + \xi\sqrt{\Omega^2 - p_i^2} + \tilde{\sigma}_{b_i}^2}$, where $\xi = \xi_{F_i}, \forall i$. When the users transmit with their maximum power, the term $\sqrt{\Omega^2 - p_i^2}$ becomes fixed, and consequently, higher bounds (e.g., higher ξ values) decrease the SINR.

The outage ratios for RTPC, RTPC-PP, TPC, and TPC-PP algorithms are shown in Fig. 11. With perfect CSI at the receivers, the expressions for power update for TPC and TPC-PP are given by (6) and (12), respectively. Note that under imperfect CSI, since the users (both PUs and SUs) need to increase their transmit powers to overcome the impact of uncertainty, which causes more interference, the zero outage for PUs in RTPC-PP is not guaranteed. Under uncertain CSI, RTPC-PP (RTPC) outperforms TPC-PP (TPC) since TPC-PP (TPC) does not consider any channel uncertainties in power updates. Note that, RTPC-PP (TPC-PP) always outperforms TPC-PP (TPC) in terms of PU outage. Since TPC does not provide any protection for PUs, under uncertain CSI, the SUs increase their transmit powers to achieve their target-SINR. This leads to zero outage for SUs but significantly increases the outage of PUs. In addition, under perfect CSI, TPC and TPC-PP do not

[3]Recall that under uncertain CSI, the power update expression for TPC-PP (TPC) is given by fourth (third) row of Table II.

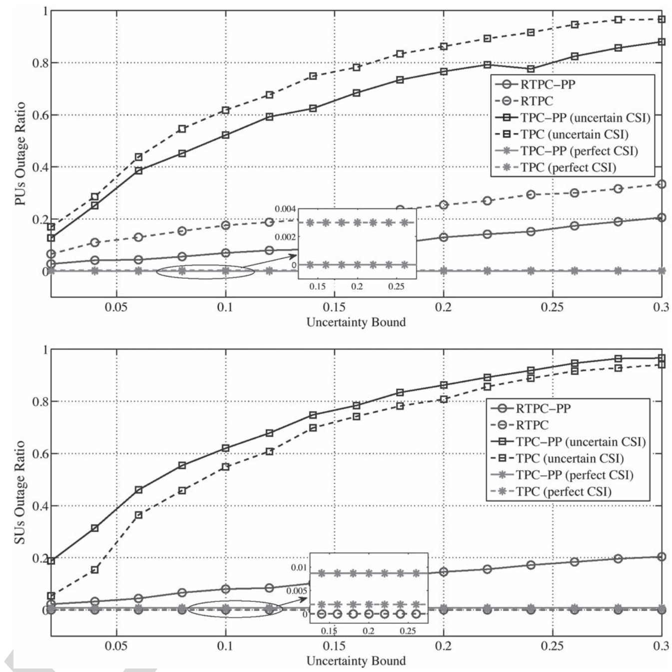


Fig. 11. Outage ratio versus uncertainty bound in RTPC, RTPC-PP, TPC, TPC-PP algorithm under perfect and imperfect CSI.

consider the channel variations, and the outage is independent of uncertainty bounds. With the perfect CSI values, the outage for PUs is always zero for TPC-PP at the cost of a higher outage for SUs, when compared to TPC.

Higher uncertainty bounds make the system more robust against channel fluctuations. However, as we have seen from Figs. 9–11, there is a trade-off between robustness and system performance since higher uncertainty bounds degrade the SINR and may increase the outage significantly.

IX. Conclusion

We have proposed distributed uplink power control algorithms (TPC-PP and ITPC-PP) for CRNs in multi-cell environments where the outage ratio for the SUs is minimized subject to the constraint of zero-outage ratio for the PUs. We have showed that our proposed distributed power-update functions corresponding to TPC-PP and ITPC-PP have at least one fixed-point. We have also showed that our proposed algorithms not only guarantee the zero-outage ratio for the PUs, but also enable the SUs to use a mixed-strategy adaptively for spectrum access to improve their outage ratio. Also, the performance of the proposed distributed ITPC-PP algorithm has been shown to be comparable to that of centralized LGR algorithm. However, the complexity of ITPC-PP is much lower than that of LGR. We have also provided a power control scheme (RTPC-PP) which provides robustness against channel uncertainties at the cost of a higher outage ratio compared to TPC-PP.

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